Vortex Avalanches in Neutron Stars

Gary (I-Kang) Liu, Andrew Baggaley, Carlo Barenghi and Toby Wood
School of Mathematics, Statistics and Physics, Newcastle University, Newcastle upon Tyne, NE1 7RU, UK

Rotational glitches can result from the angular momentum exchange between the crust and neutron star (NS) interiors. We study the dynamics of about 600 quantum vortices in a spinning-down, two-dimensional neutron superfluid using the Gross-Pitaevskii (GP) model. For the first time, we find convincing spatial-temporal evidence of avalanching behaviour with about 10-20 vortices in each event resulting from vortex depinning and collective motion, during glitches and their post-evolutions. In the later stage, vortices continue to depin and circulate around the vorticity voids in a similar manner to that seen in previous point-vortex simulations. We also demonstrate the exponential and power-law distributions in the avalanche waiting time and size under a controllable setup.

Formalism

n the GP framework, the superfluid (SF) is described by a complex wavefunction $\psi(\mathbf{r},t)$ satisfying the GP equation [1,2],

$$i\hbar\partial_t\psi(\mathbf{r},t) = (1 - i\gamma)\left[-\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r},t) + g|\psi(\mathbf{r},t)|^2 - \Omega(t)\hat{L}_z - \mu\right]\psi(\mathbf{r},t)$$

where $m = 2m_n$ is the mass of SF particle (a neutron Cooper pair), γ is the dissipation rate, g describes the self-repulsion of the SF, and μ is the effective chemical potential giving the characteristic length and time scales, $\xi = \hbar/\sqrt{m\mu}$ and $\tau = \hbar/\mu$. Here we consider the imposed potential by

$$V(\mathbf{r},t) = V_{\rm con}(\mathbf{r}) + V_{\rm pin}(\mathbf{r}) + \frac{1}{2}m\Omega(t)^2(x^2 + y^2),$$

where $V_{\rm con}(\mathbf{r})$ is a cylindrical hard-wall confinement with a radius 231ξ and $V_{\rm pin}(\mathbf{r}) = \sum_k 2\,\mu\exp\left[-(\mathbf{r}-\mathbf{r}_k)^2/\xi^2\right]$ with $\mathbf{r}_k$ in square lattice setup. A linearly spinning-down crust due to the electromagnetic brake is considered by

$$\Omega(t) = \Omega_0 - \dot{\Omega}t.$$

Numerical Results

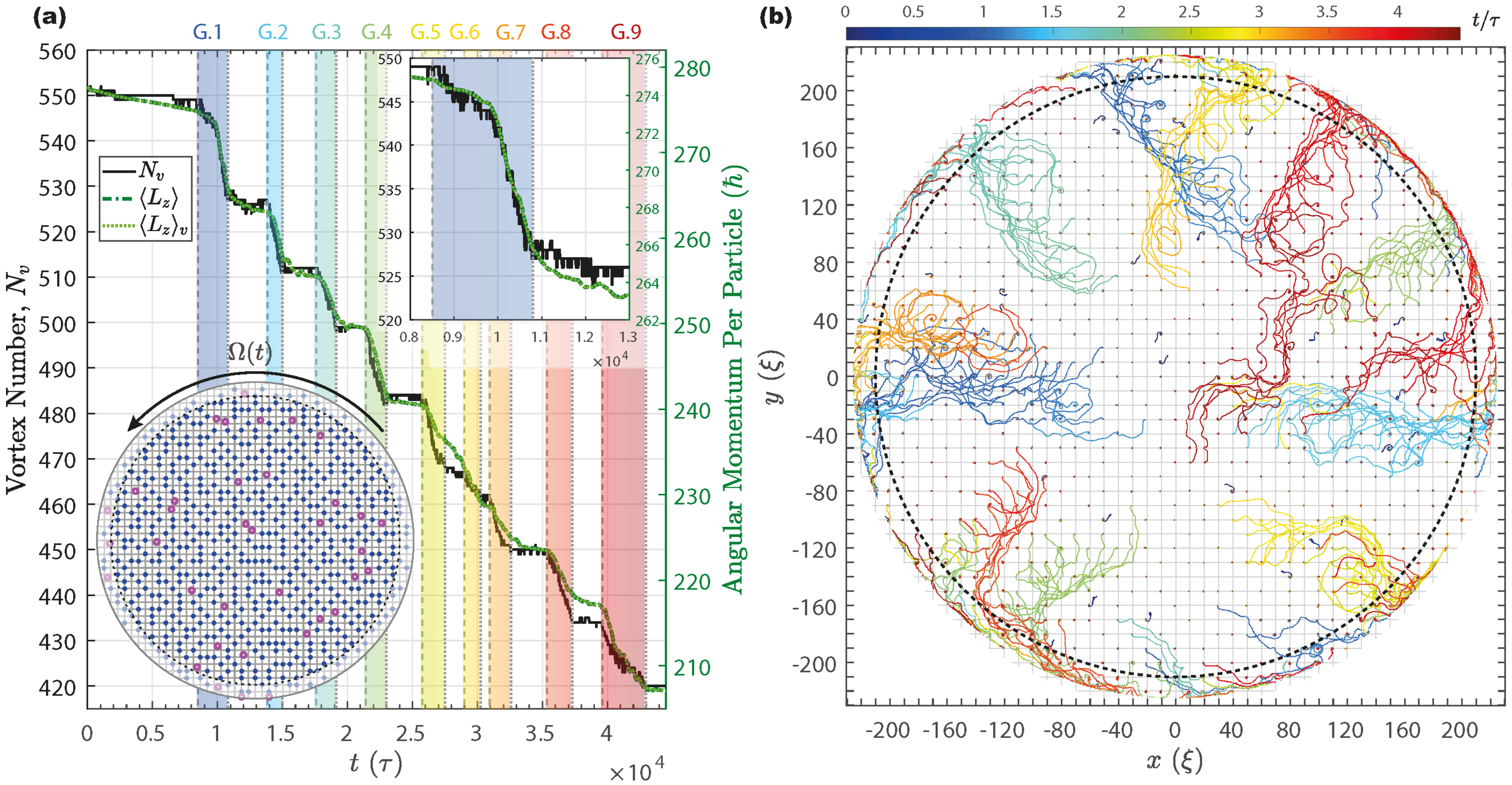

Figure 1: (a) Glitches in the superfluid rotation rate for $\Omega_0 = 4\pi\times10^{-3}\tau^{-1}$, $\dot{\Omega} = 2.5\pi\times10^{-8}\tau^{-2}$ and $\gamma = 5\times10^{-3}$. The number of vortices, N_v, is closely correlated with the angular momenta per particle, $\langle L_z\rangle = \int_A d\mathbf{r}\psi^*\hat{L}_z\psi \,/\int_A d\mathbf{r}|\psi|^2$, and the Fetter approximation $\langle L_z\rangle_v = \hbar\sum_j\left[1-|\mathbf{x}_j|^2/R^2\right]$ [3] with the circular area A within $R = 210\xi$. The bottom-left inset shows the initial vortex configuration (pinned vortices in blue and free ones in red), and the top-right one shows a close-up of the first glitch (G.1). (b) Vortex trajectories in the rotating frame, showing that the glitches arise from **vortex avalanches**.

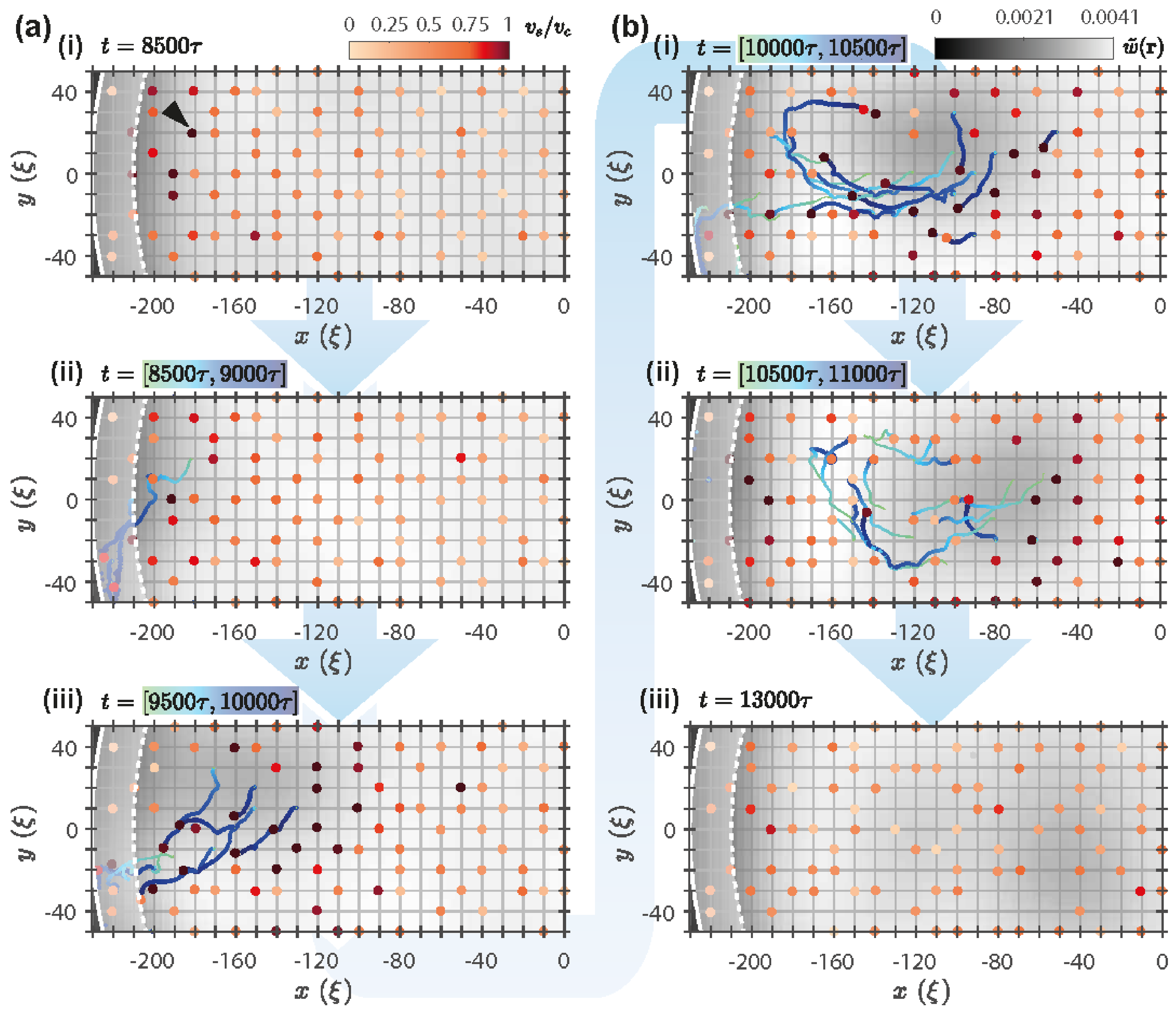

Figure 2: The vortex dynamics for (a) the first glitch (b) the subsequent **post-glitch** evolution. The vortices in each snapshot are colored by the vortex velocity scaled by an empirical critical velocity $v_c = 0.2\sqrt{\mu/m}$ at which depinning occurs [4]. The vortex velocity is estimated by the point-vortex approach [5],

$$\mathbf{v}_{s,j} = \sum_{j\neq k}\frac{\hat{\mathbf{z}}\times[\vec{r}_j(t) - \vec{r}_k(t)]}{|\vec{r}_j(t) - \vec{r}_k(t)|^2} - \Omega(t)\vec{r}_j(t)$$

The vortex trajectories over the time span of $500\tau_0$ are shown in the light blue to dark blue color gradient. The background is the coarsened vorticity distribution [6] in greyscale.

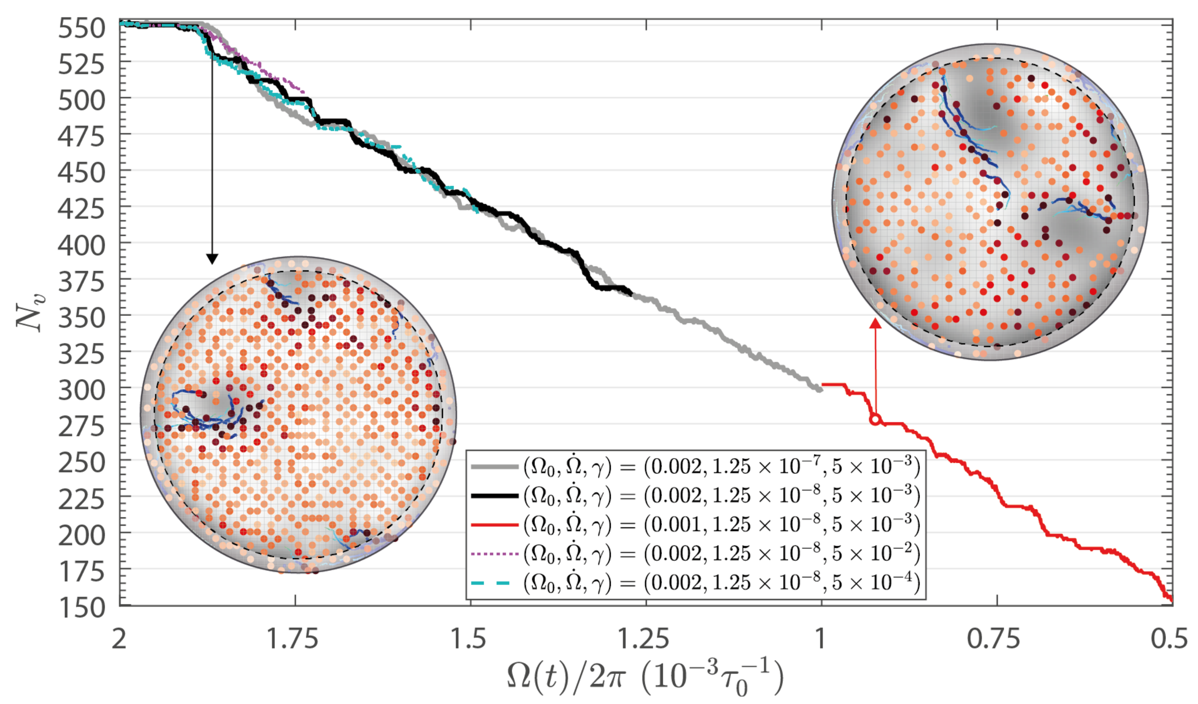

Figure 3: The time series of N_v as a function of $\Omega(t)$ for different parameters. The disk insets show the vortex trajectories and vorticity distribution immediately after the first glitch for two different vortex densities. For smaller Ω_0 we observe similar dynamics, but with larger **voids in the vorticity distribution**.

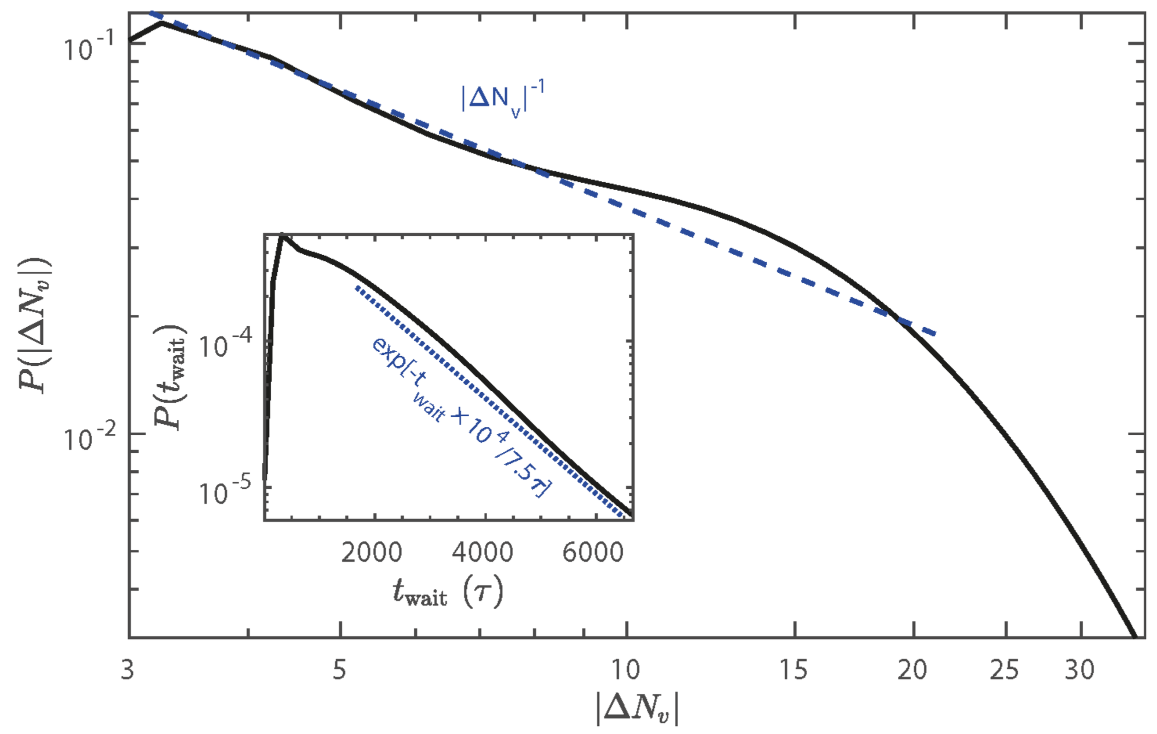

Figure 4: The kernel density of the avalanche size ΔN_v over more than 320 events from 30 individual simulations, showing a clear **power-law distribution**. The inset shows the kernel density of waiting time between avalanches in log-lin scale, highlighting the **exponential distribution**.

Remarks

1. Our 2D GP model with the **largest number of vortices**, ≈ 600, **smallest dissipation** and **spinning-down rate** to date demonstrates vortex avalanche dynamics.
2. The vortex avalanches are associated with self-organized criticality (featuring **power-law** and **exponential distributions** in glitch size and waiting time) [7]. Each avalanche has 3 stages:
   (a) Trigger: A few vortices unpin and migrate out after they experience thresholds of net shear flows.
   (b) Avalanche: more unpinned vortices are elicited from the trigger and collectively move outward.
   (c) Post-relaxation: The vortex avalanche leaves voids in the (coarsened) vorticity distribution, and vortices around the voids unpin and orbit anti-rotational axis-wise around them to redistribute vortices until the system is in a semi-equilibrium state.
3. The glitch-like drops in both of vortex number and angular moment are **more apparent** in the simulation with **lower spin-down rate** $\dot{\Omega}$.
4. The voids in vorticity distribution become larger as vortices get further apart, which can be up to meter scale in NSs.

References

[1] L. Warszawski & A. Melatos, MRAS 415, 1611(2011).
[2] I.-K. Liu et al., ApJ 984, 83 (2025)
[3] A. Fetter, Physical Review, 138, 5 (1965).
[4] I.-K. Liu et al., JLTP 215, 376 (2024)
[5] A. Baggaley et al. , PRL 109, 205304 (2012).
[6] G. Howitt et al., MNRAS 498, 320 (2020).
[7] A. Melatos et al., ApJ 672, 1103 (2008).