STATISTICAL THEORY OF THREE-BODY INTERACTIONS IN CLUSTERS

BARRY GINAT

LEVERHULME-PEIERLS FELLOW, DEPARTMENT OF PHYSICS & NEW COLLEGE, OXFORD

WITH HAGAI PERETS, DANY ATALLAH & NEWLIN WEATHERFORD

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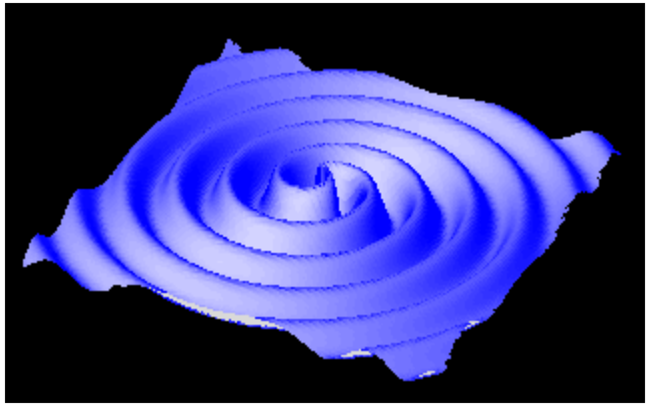
Based on: 2011.00010, 2108.01085, 2205.15957, 2503.14605

UNIVERSITY OF OXFORD

THREE-BODY INTERACTIONS ARE EVERYWHERE IN ASTROPHYSICS

M80 (Hubble)



http://lisa.jpl.nasa.gov/IMAGES/wavy.gif

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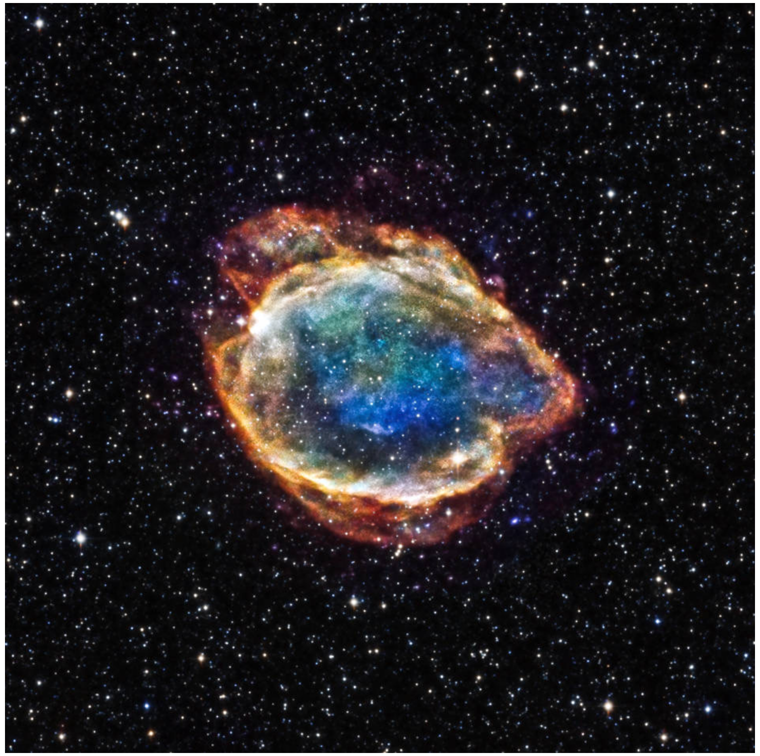

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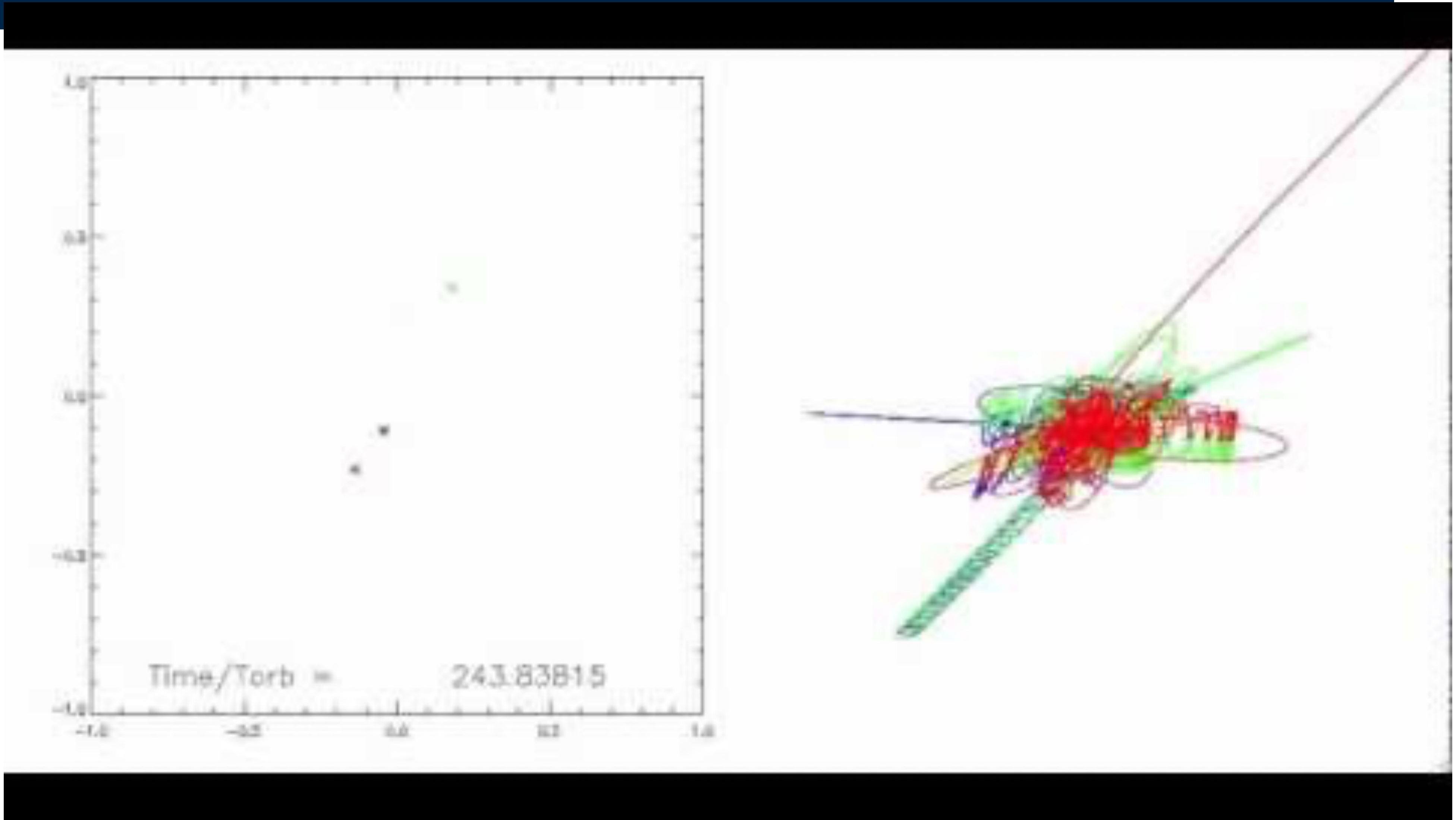
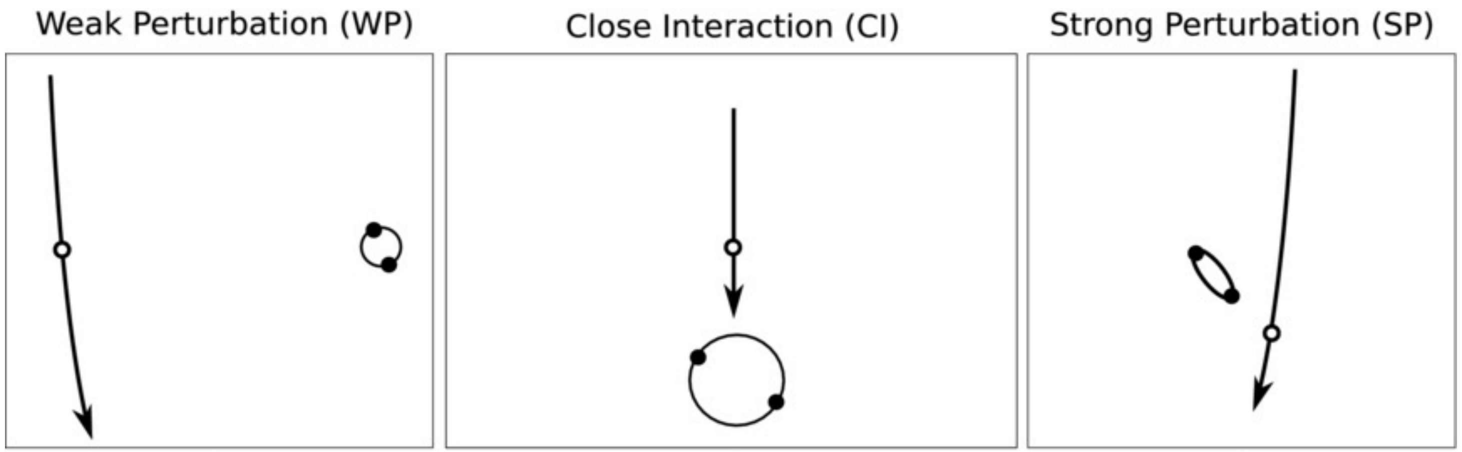
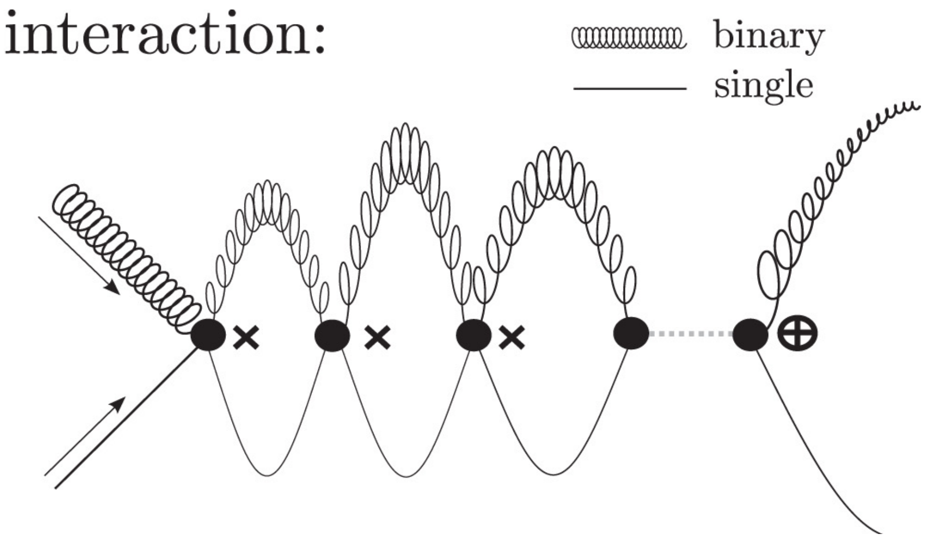
NUMERICAL SIMULATIONS NOW ALLOW SCIENTISTS TO SOLVE



Samsing et al. (2014), http://youtu.be/ipPniBvZvxY

CLASSIFICATION OF BINARY-SIGNLE ENCOUNTERS



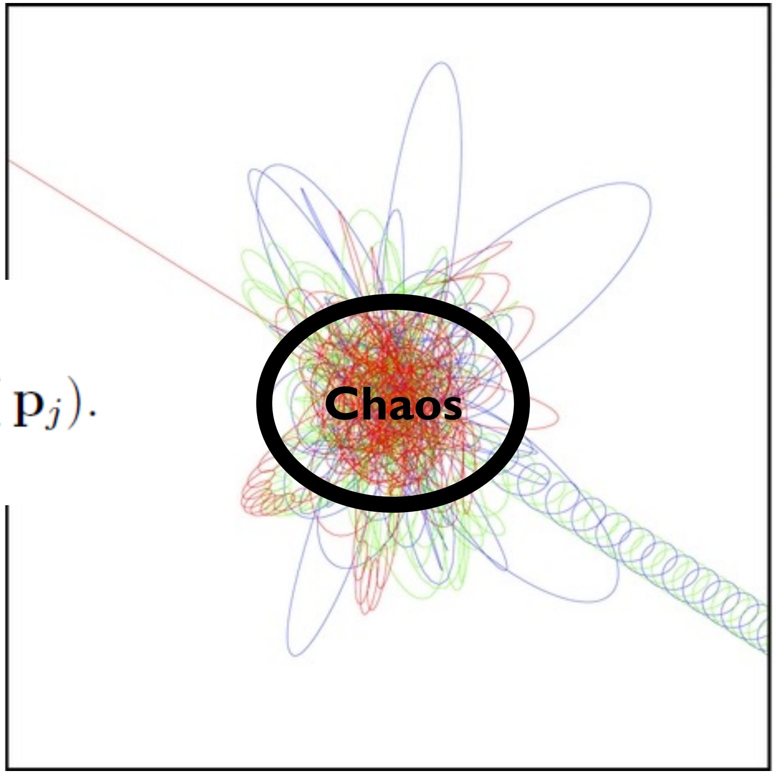


Samsing et al. (2014,2017)

THE TOTAL CROSS-SECTION IS THE PHASE-SPACE VOLUME OF THE CHAOTIC REGION

Interesting quantity: the phase-space volume of the chaotic region (Monaghan 1974,…).

$$\sigma = \left(\prod_{i=1}^{3}\int_{\mathcal{C}} \mathrm{d}^3\mathbf{r}_i \mathrm{d}^3\mathbf{p}_i\right)\delta(E-\mathscr{H})\delta(\mathbf{J}-\sum_{j=1}^{3}\mathbf{r}_j\times\mathbf{p}_j)\delta(\mathbf{P}_{\mathrm{CoM}}-\sum_{j=1}^{3}\mathbf{p}_j).$$



σ is computed using angle-action variables (Stone & Leigh 2019).

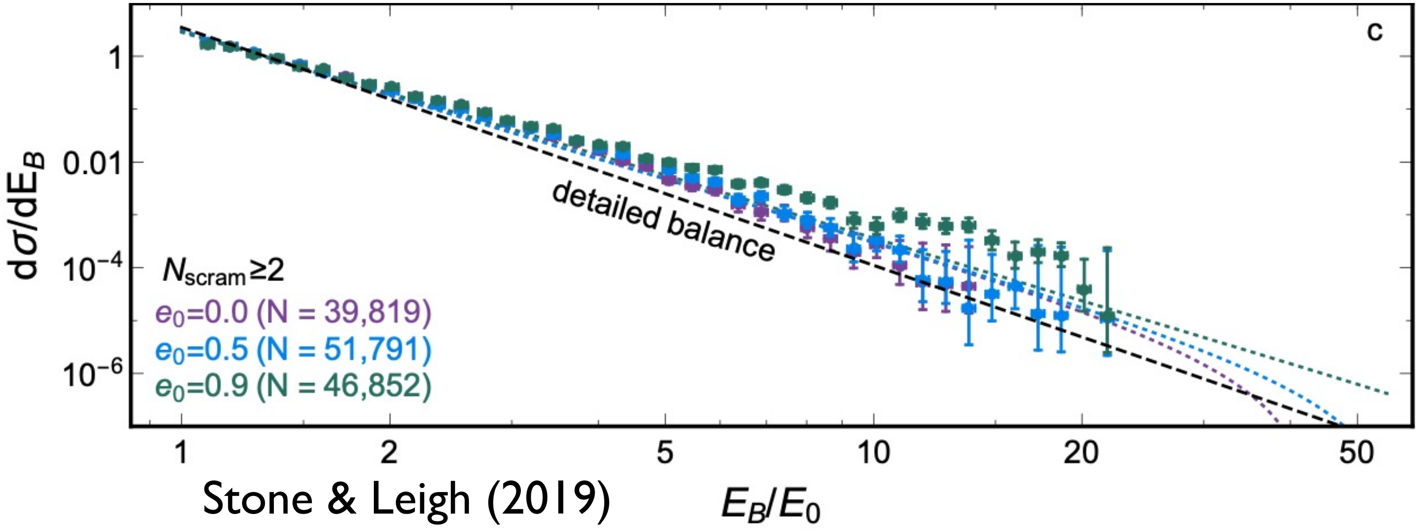
- Compare with constant flux theory (Kol 2021).

UNBOUND CROSS SECTION

- Stone and Leigh (2019) found f_{bin} proportional to:

$$f_{bin}(E_{bin}, \boldsymbol{S}|E, \boldsymbol{J}) \propto \frac{\theta_{\max}}{|\boldsymbol{J}-\boldsymbol{S}||E_0 - E_{bin}|^{\frac{3}{2}}|E_{bin}|^{\frac{3}{2}}}$$

- For the bound (Ginat & Perets 2021) need to specify integration region explicitly.

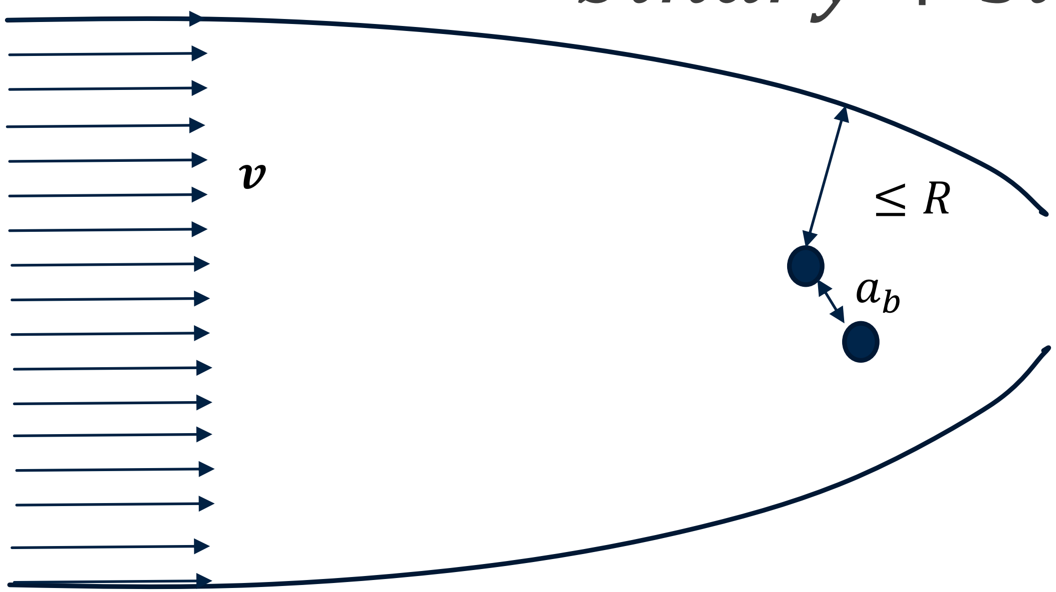


Stone & Leigh (2019)

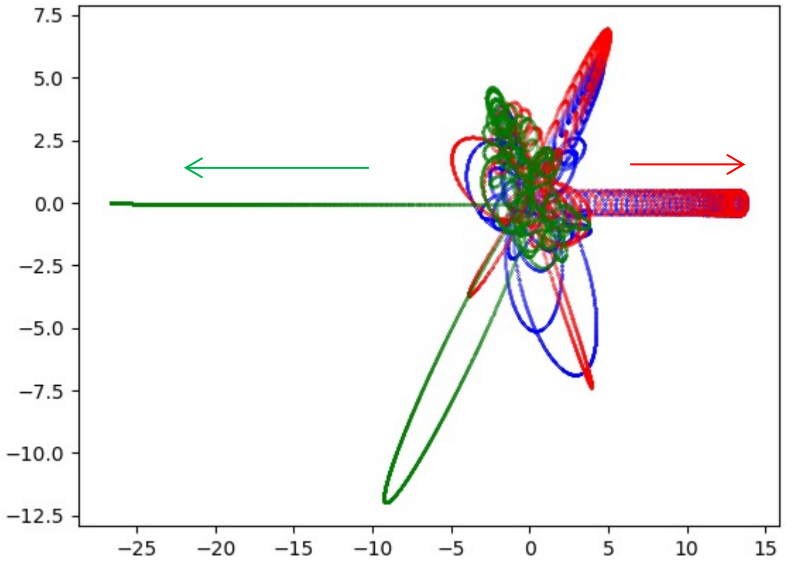
THE PRINCIPLE OF DETAILED BALANCE

- By the principle of detailed balance (Heggie 1975 and more)

$$binary + single \rightleftharpoons triple$$



$\rightleftharpoons$



THE PRINCIPLE OF DETAILED BALANCE

- Then, by the principle of detailed balance (Ginat & Perets 2021)

$$Triples \rightleftharpoons binary + single$$

$$n_{\mathrm{triple}}(E_0,\mathbf{J})\Gamma(E_0,\mathbf{J}\to E_{\mathrm{bin}},\mathbf{S}) = \int \mathrm{d}^3\mathbf{R}_{\mathrm{cm}}\mathrm{d}^3\mathbf{P}\mathrm{d}^3\mathbf{p}\delta(\mathbf{R}_{\mathrm{cm}})\delta(\mathbf{p}_{\mathrm{bin}}+\mathbf{p}_3)\times n(\mathbf{p}_3)n_{\mathrm{bin}}(E_{\mathrm{bin}},\mathbf{S})\Gamma(E_{\mathrm{bin}},\mathbf{S},\mathbf{p}\to E_0,\mathbf{J})$$

Disintegration

Formation

THE RATE OF DISINTEGRATION

- The rate of disintegration is simply

$$\Gamma(E_0, \boldsymbol{J} \ \to E_{bin}, \boldsymbol{S}) \propto \Omega_c^s f_{bin}(E_{bin}, \boldsymbol{S} | E_0, \boldsymbol{J}).$$

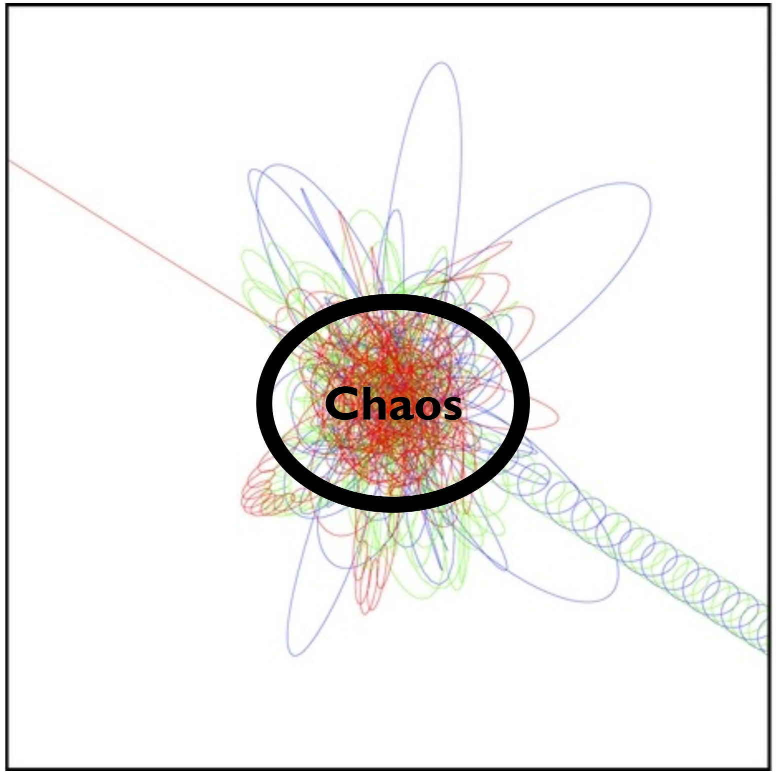
- But by detailed balance

$$\Gamma(E_0, \mathbf{J} \to E_{\text{bin}}, \mathbf{S}) \sim \frac{\rho^3 e^{-\beta^* E_0} (m_1 m_2 m_3)^{-3/2} m_{\text{bin}} \mu_{\text{bin}}^{3/2}}{n_{\text{triple}}(E_0, \mathbf{J}) E_{\text{bin}}^{3/2} |\mathbf{J} - \mathbf{S}|}$$

THE STRONG-INTERACTION REGION IS DEFINED BY THE HIERARCHICAL LIMIT

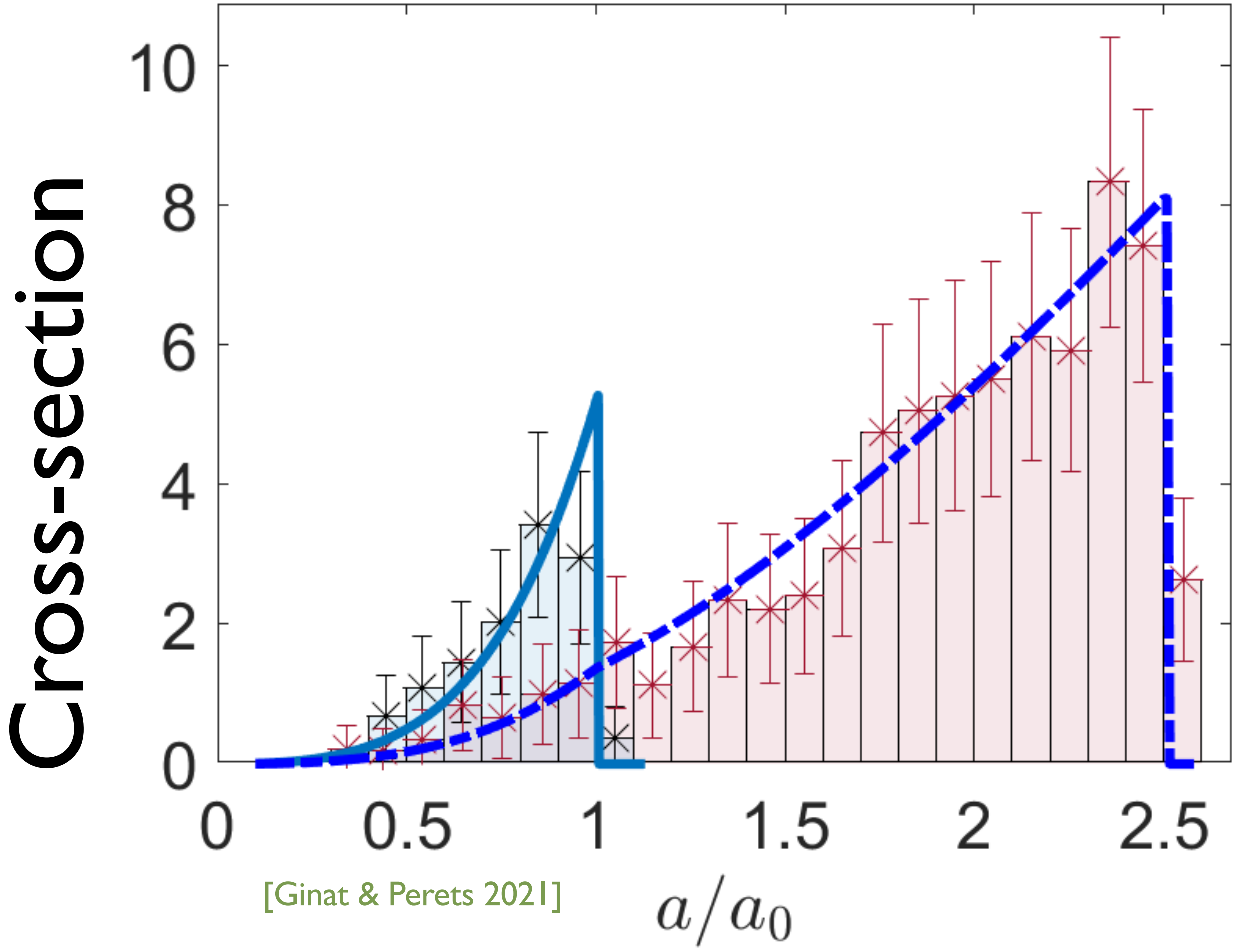
- Need: region in phase-space where system is far from being hierarchical.
- Equate quadrupole and monopole (Ginat & Perets 2021)

$$R = \beta \min\left\{ \left(\frac{G\mu_{\rm bin}\mu_s M}{m_{\rm bin}\,|E|} \right)^{1/3} a_{\rm bin}^{2/3},\, a_{\rm bin} \right\}$$



EXAMPLE: ANALYTIC RECONSTRUCTION OF NUMERICAL RESULTS – FINAL SEMI-MAJOR AXIS DISTRIBUTION

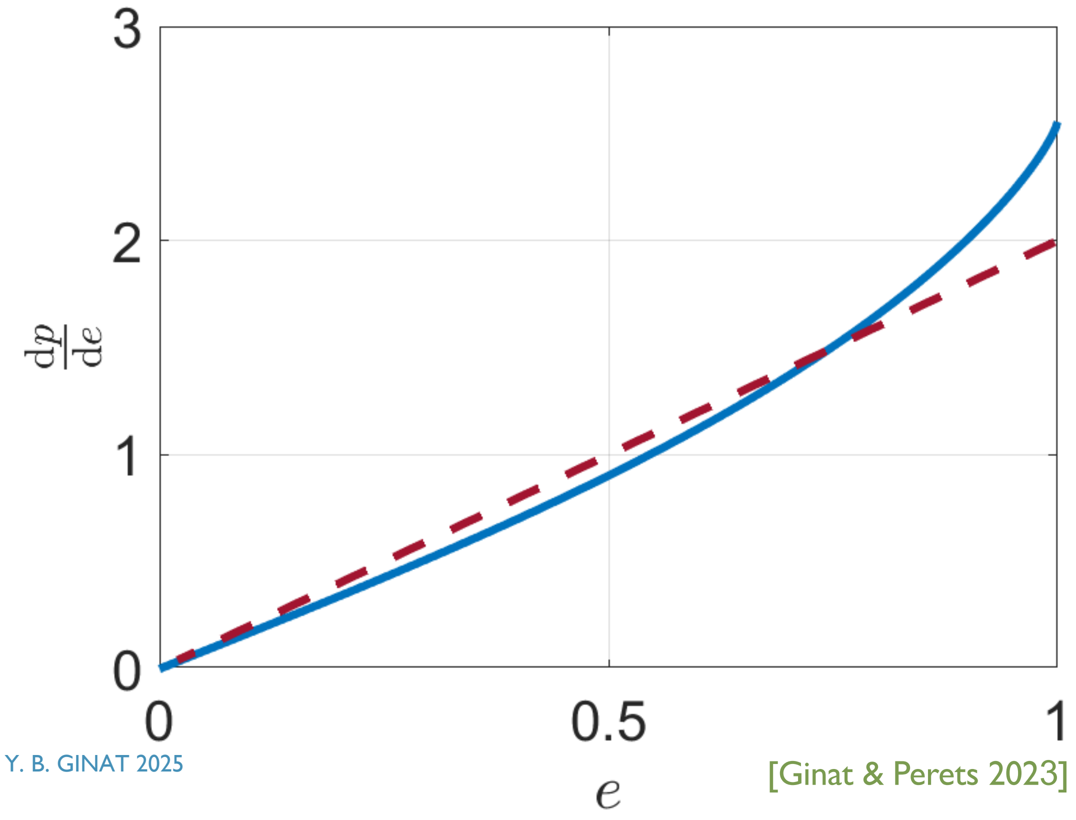
Data from: Sigurdsson & Phinney (1993)

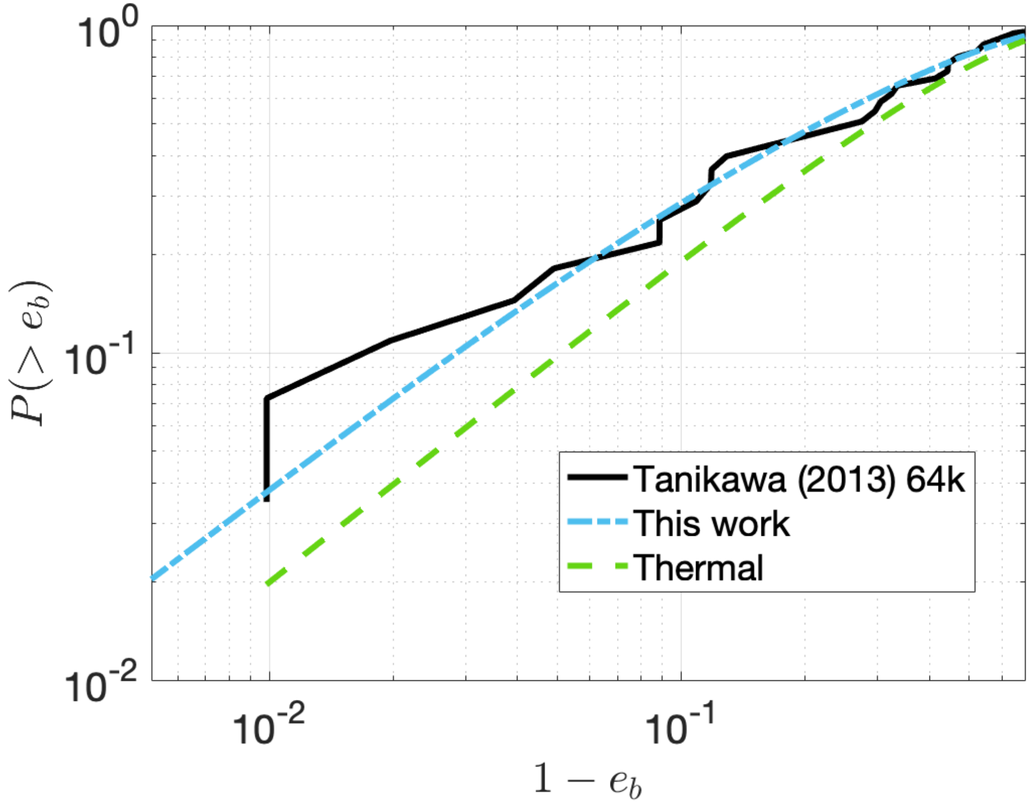


[Ginat & Perets 2021]

ECCENTRICITY OVER MULTIPLE ENCOUNTERS

What is the probability of finding a given eccentricity in a randomly chosen hard binary in a cluster?

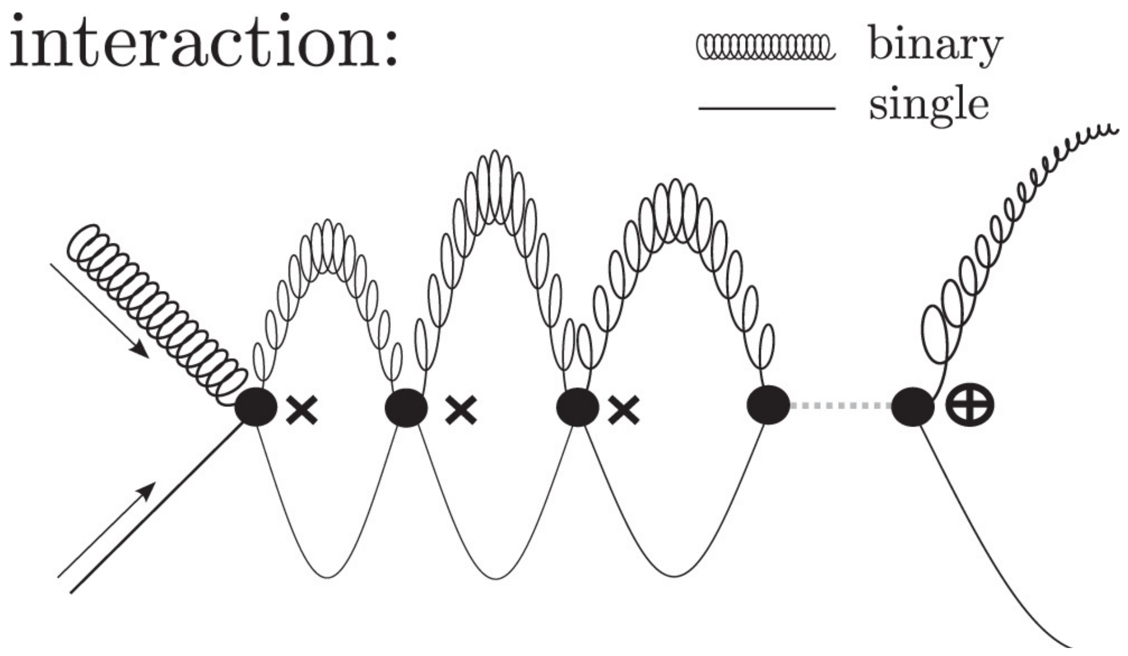




[Ginat & Perets 2023]

THE ENTIRE ENCOUNTER IS MODELLED BY A RANDOM WALK FROM ONE CLOSE APPROACH TO THE NEXT

- E_b and $\boldsymbol{S}$ are randomly selected in each step.
- Dissipation changes E and J and so distribution of orbital parameters.



DISSIPATION INDUCES A CONVEX COMBINATION OF f_{bin}'S

- Let $x = (E_{bin}, \mathbf{S}, E)$. Then the final distribution is

$$P(x) = f_{\mathrm{bin}}(E_{\mathrm{bin}}, \mathbf{S}|E, \mathbf{J}) \int \mathrm{d}\tilde{\lambda}\tilde{p}(\tilde{\lambda}|E_0)\delta(E - (E_0 - \tilde{\lambda}))$$

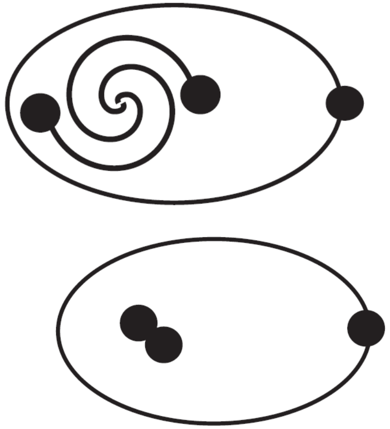
Single-step distribution, given total energy E

Probability for change λ in E due to dissipative process

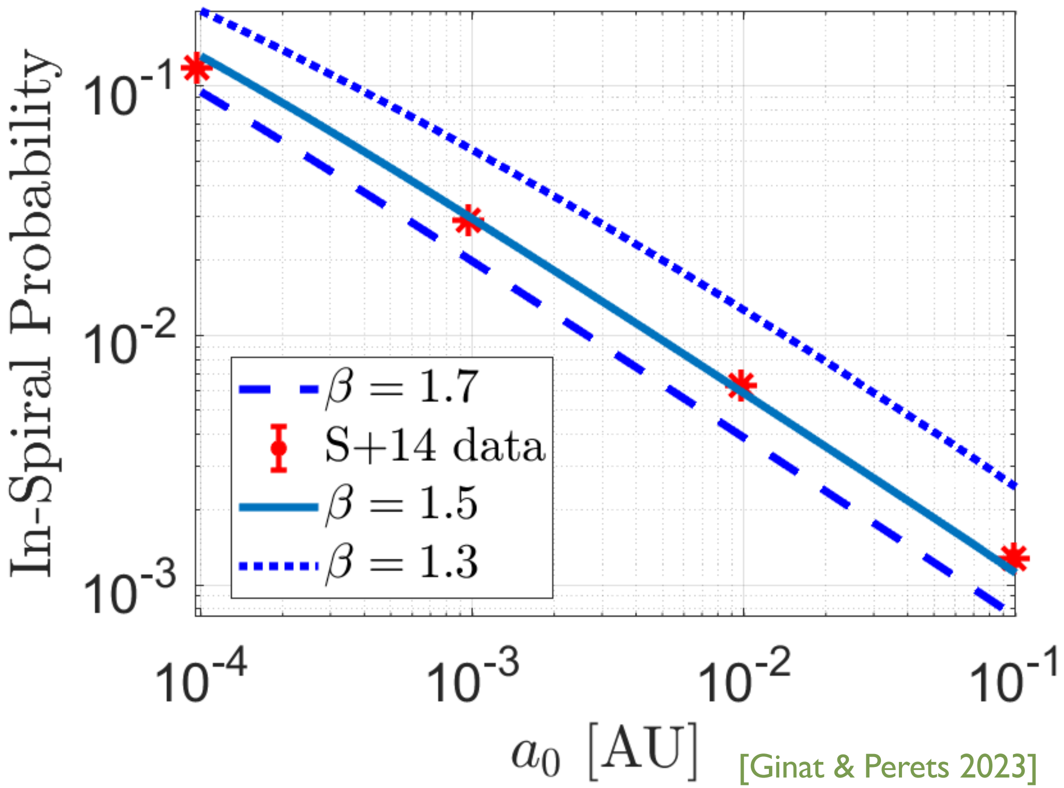
Initial total energy

GRAVITATIONAL-WAVES SOURCES

- 3 channels:
  - Binary hardening
  - Inner binary in a hierarchical phase.
  - “Collision” in chaotic phase



$m_1 = m_2 = m_3 = 1M_{\odot}, v_0 = 10$ km/sec



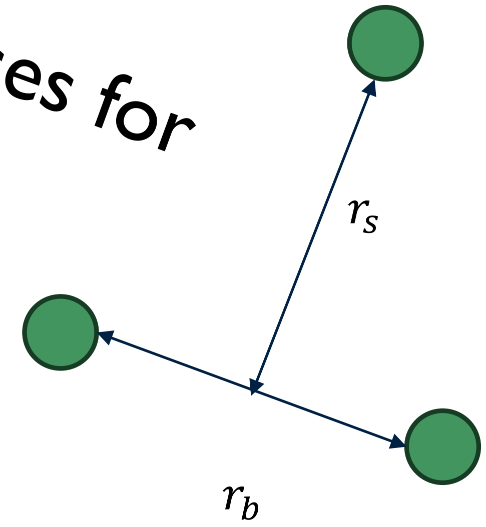
[Ginat & Perets 2023]

Data from: Samsing et al. (2014)

CAN WE DO THE SAME FOR POSITIVE ENERGY SYSTEMS?

- Democratic resonances are unlikely to last long.
- But properties of resultant binaries still sensitive to initial conditions.
- So hypothesise that phase-space mixing assumption still holds for an ensemble.
- Cf. Aarseth & Heggie (1976), Goodman & Hut (1993), …

Equal masses for now!



(most bound/ least unbound)

UNBOUND TRIPLE ENCOUNTERS FORM BINARIES

- One can use the density of states formalism to calculate the rate of three-body binary formation.

$$\frac{\mathrm{d}P}{\mathrm{d}a_b \mathrm{d}e} = \frac{1}{P_{\mathrm{bin}}} \left[\frac{m_1 m_2 m_3}{M} \right]^{3/2} \frac{V_0^{-2}}{[2\pi k_{\mathrm{B}} T]^3} \int \mathrm{d}^3 r \mathrm{d}^3 R \mathrm{d}^3 v \mathrm{d}^3 V \times \mathrm{e}^{-\frac{E}{k_{\mathrm{B}} T}} f_{\mathrm{bd}}(E_b, S | E, J; R_0) \frac{\mathrm{d}E_b}{\mathrm{d}a_b} \frac{\partial S}{\partial e_b},$$

- Here

$$f(E_b, \boldsymbol{S} | E, \boldsymbol{J}) \propto m_b \frac{\theta_{\max}(R_0, E_b, S) \theta_{\max}(R_s, E_s, |\boldsymbol{J} - \boldsymbol{S}|)}{|\boldsymbol{J} - \boldsymbol{S}| E_s^{3/2} |E_b|^{3/2}}$$

UNBOUND TRIPLE ENCOUNTERS FORM BINARIES

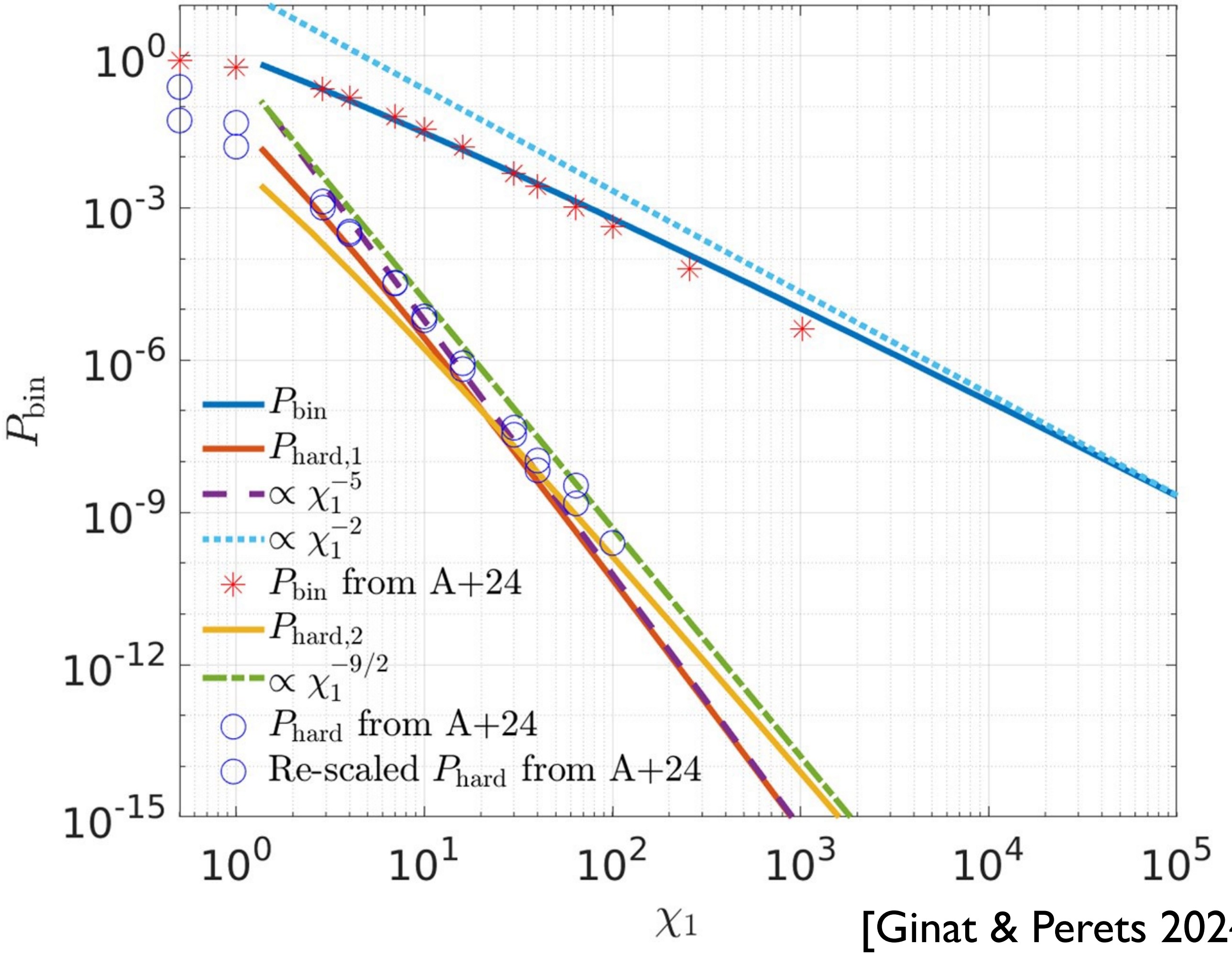
- Rate is

$$\Gamma \propto R_1^5 n^3 \sigma \times P_{\text{bin}}$$

- Soft-binary production $\propto R_1^{-2}$ in agreement with Aarseth & Heggie (1976).
- We get:

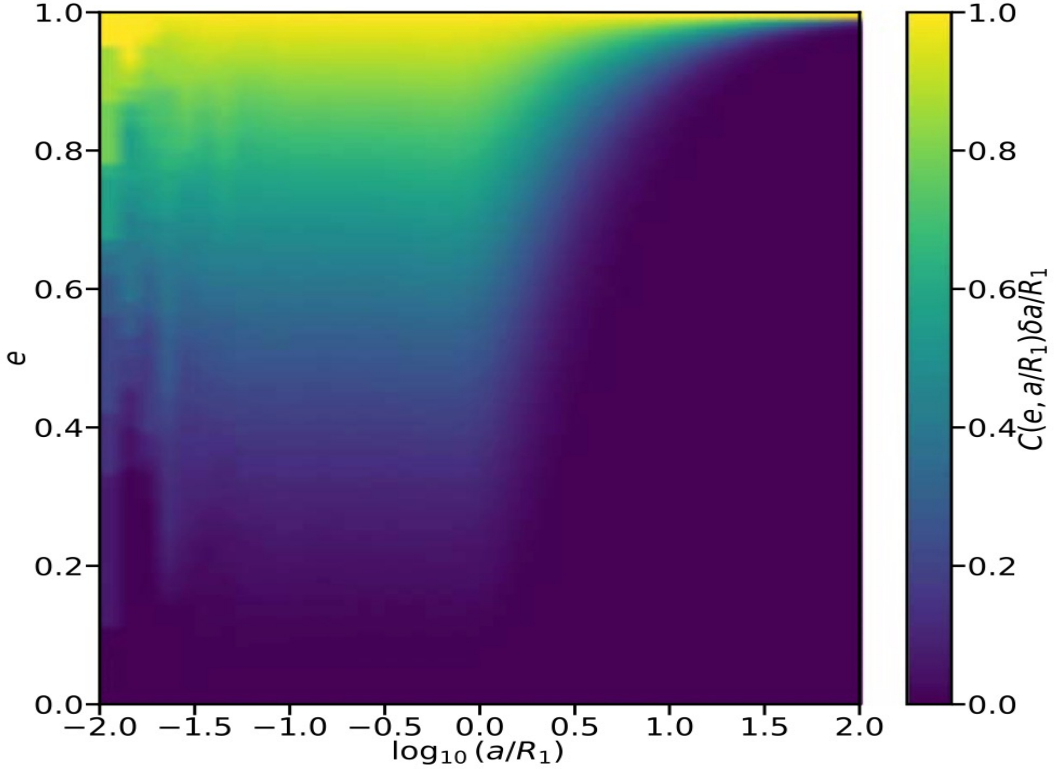
$$\Gamma = 0.68\ \Gamma_{\text{Heggie \& Hut 2003}}$$

$$\chi_1 = \frac{27 R_1 k_B T}{2GM^2}$$

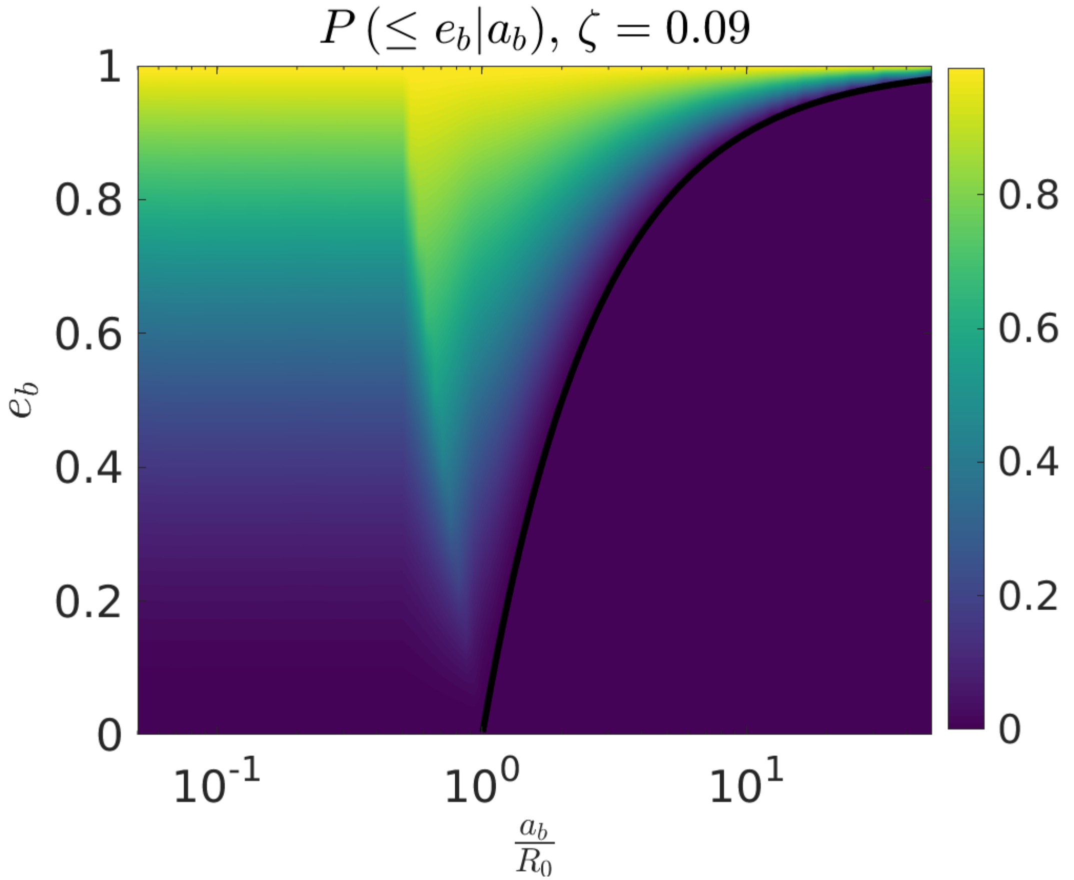


[Ginat & Perets 2024]

THESE BINARIES ARE OVERWHELMINGLY ECCENTRIC AND SOFT



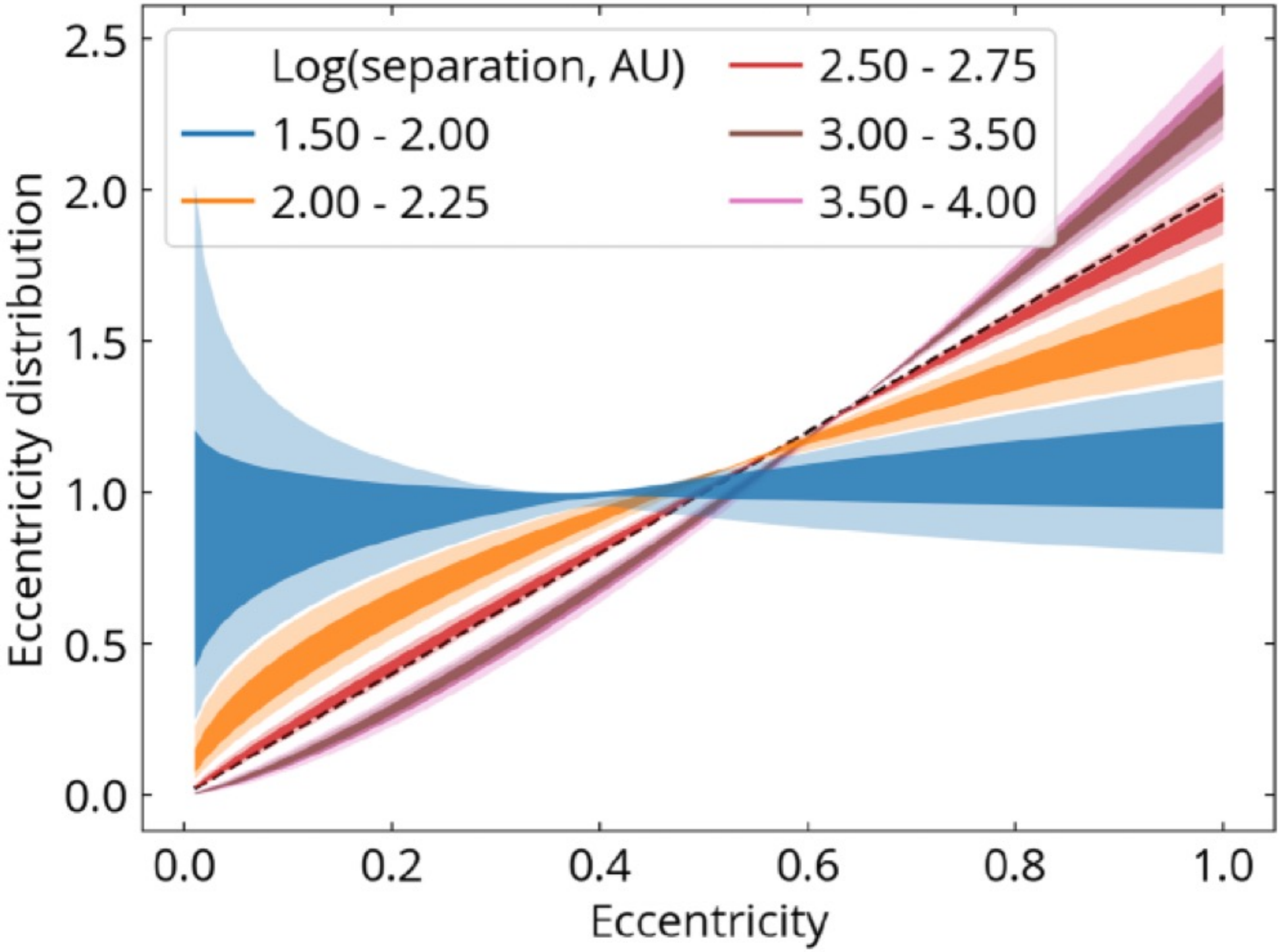
Atallah et al. (2024)



Ginat & Perets (2024)

GAIA WIDE BINARIES HAVE A SUPER-THERMAL ECCENTRICITY DISTRIBUTION

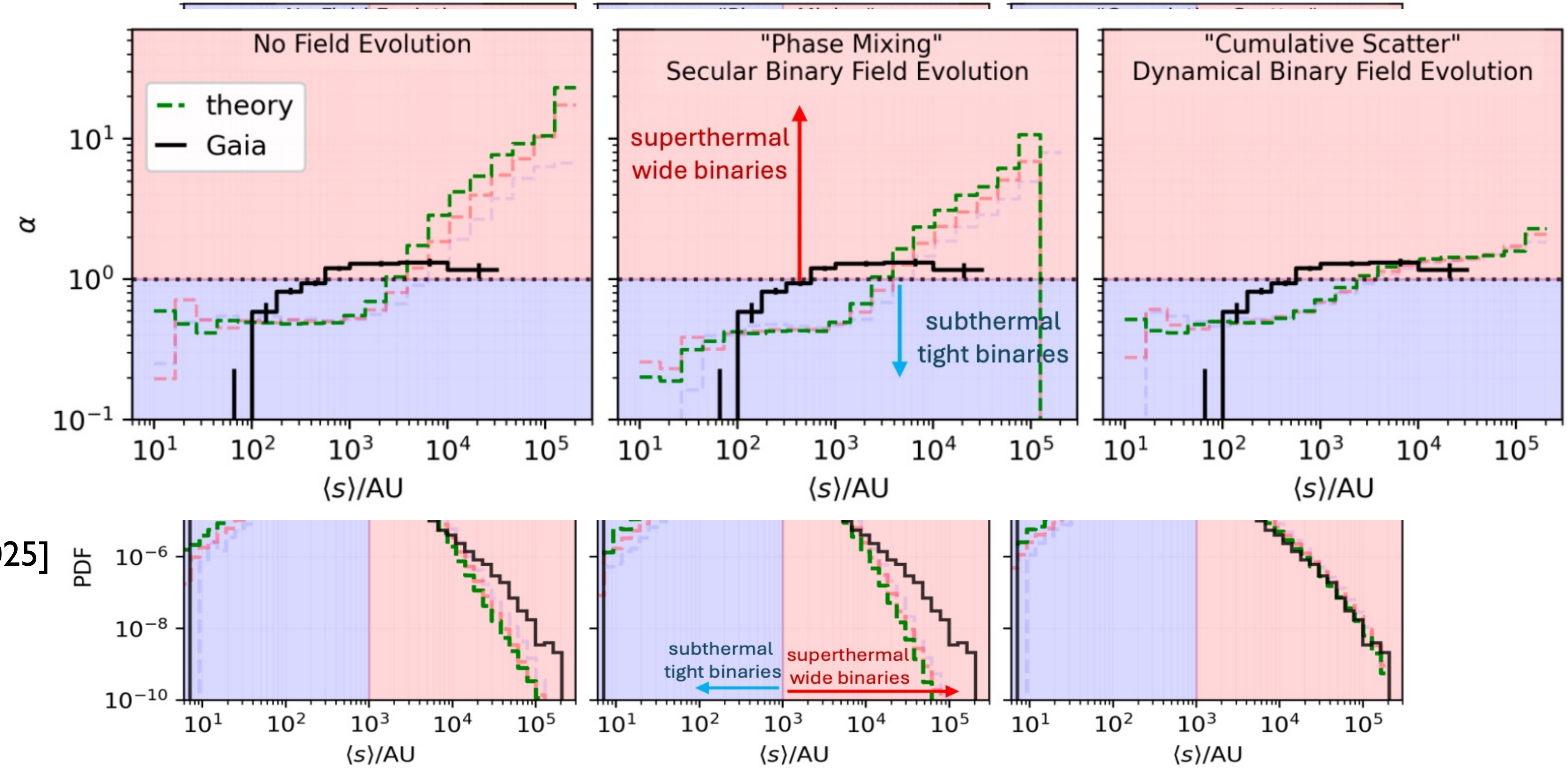
- Binaries observed by Gaia (in DR3).
- Hwang et al. (2022) found eccentricity distribution becomes super-thermal.



NEED TO ACCOUNT FOR EVOLUTION IN CLUSTER AND FIELD

- Hamilton et al. have shown that it is very difficult to get a super-thermal distribution from dynamical binary evolution via weak interactions.
- Can three-body binary formation (i.e. strong dynamical interaction) explain this?
- Copious numbers of 3-body binaries form constantly and are destroyed constantly.
- Need to account for cluster evolution before the binary leaves it:
  - Disruption by other stars
  - Survival until dissolution (i.e. forms when cluster is small or at outskirts)
  - These depend on mass and age of the cluster, which depend on Milky-Way history.
  - Field evolution (Galactic tides, encounters).

CAN THREE-BODY BINARIES EXPLAIN IT?



[Atallah et al. 2025]

SUMMARY

- Three-body interactions are ubiquitous in dense dynamical systems.
- Binaries that undergo such interactions have unique orbital parameter distributions.
- Contact me at yb.ginat@physics.ox.ac.uk