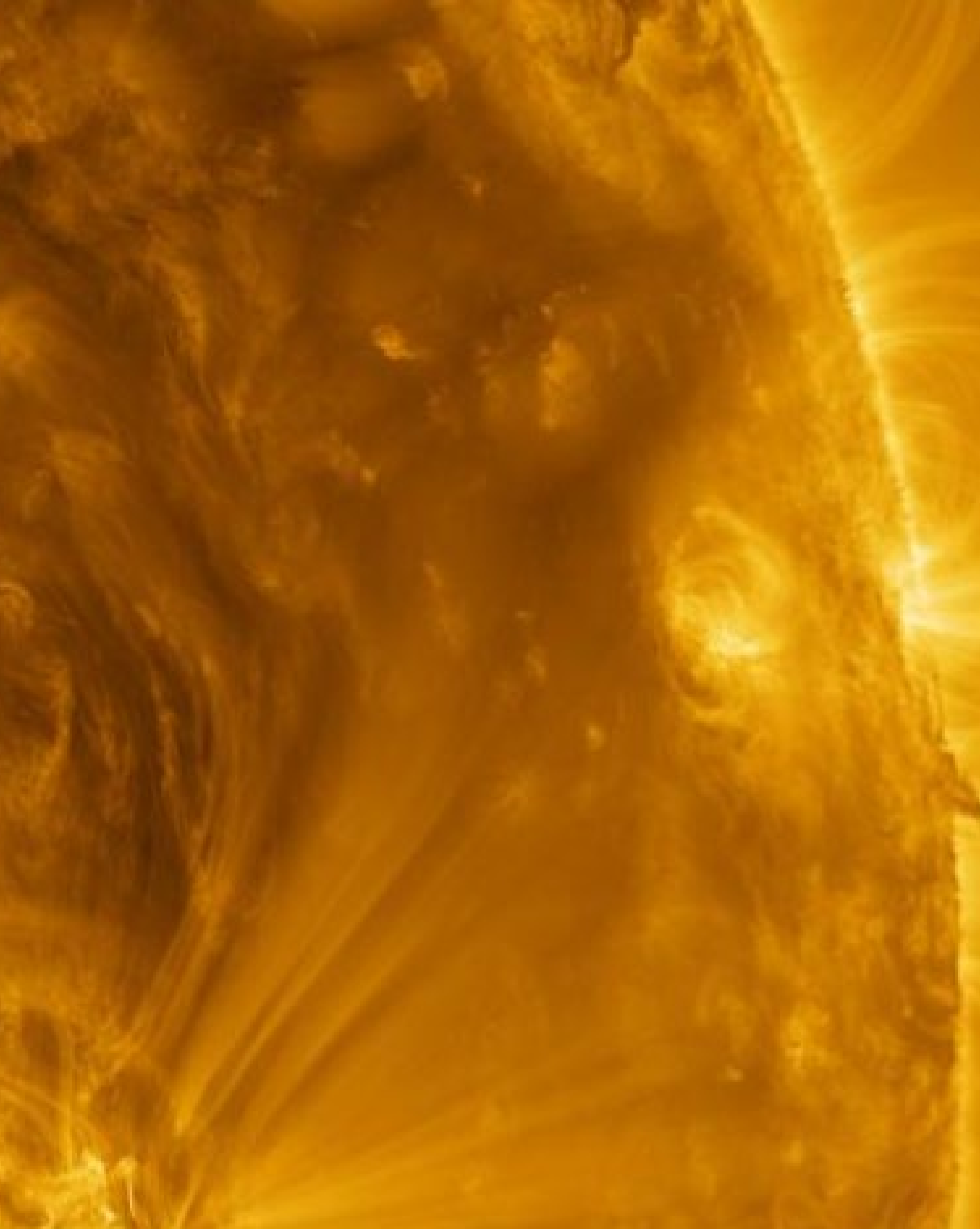

On field line slippage rates in the solar corona

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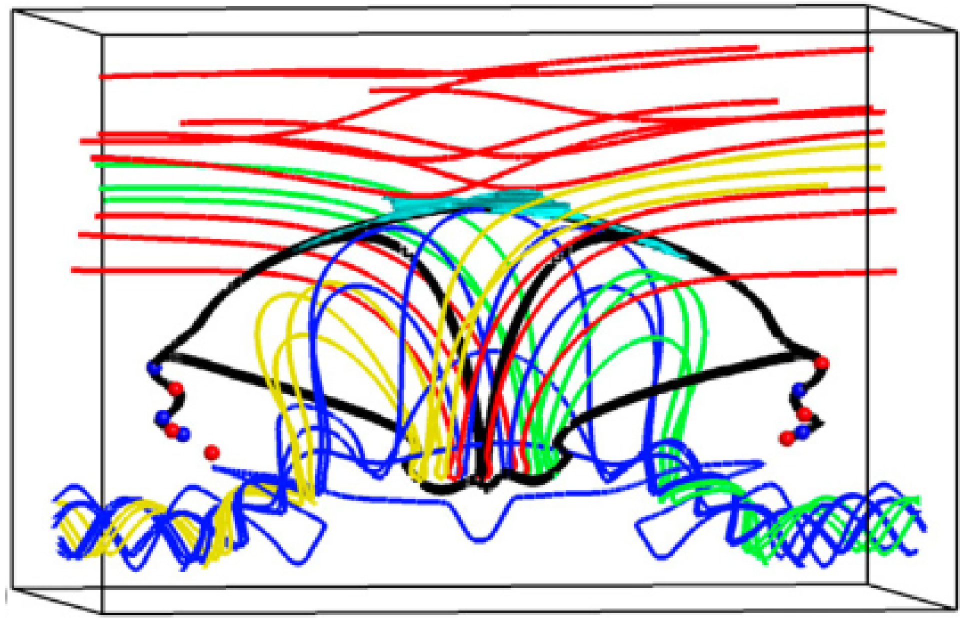
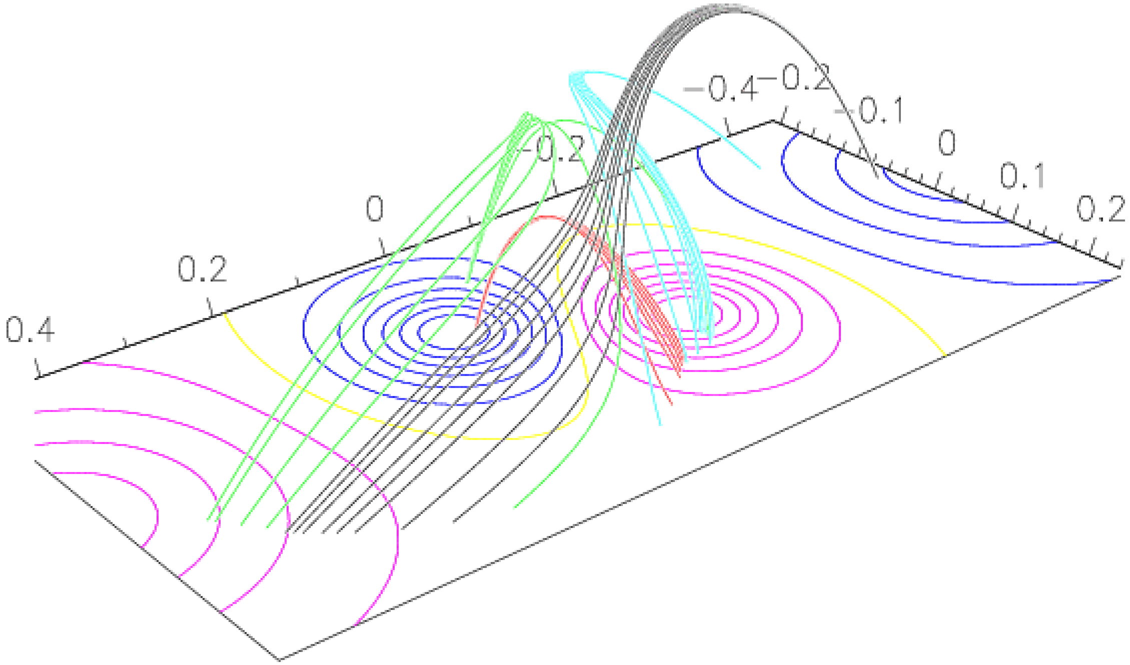
NAM 2025

Credit: Solar Orbiter

3D magnetic reconnection

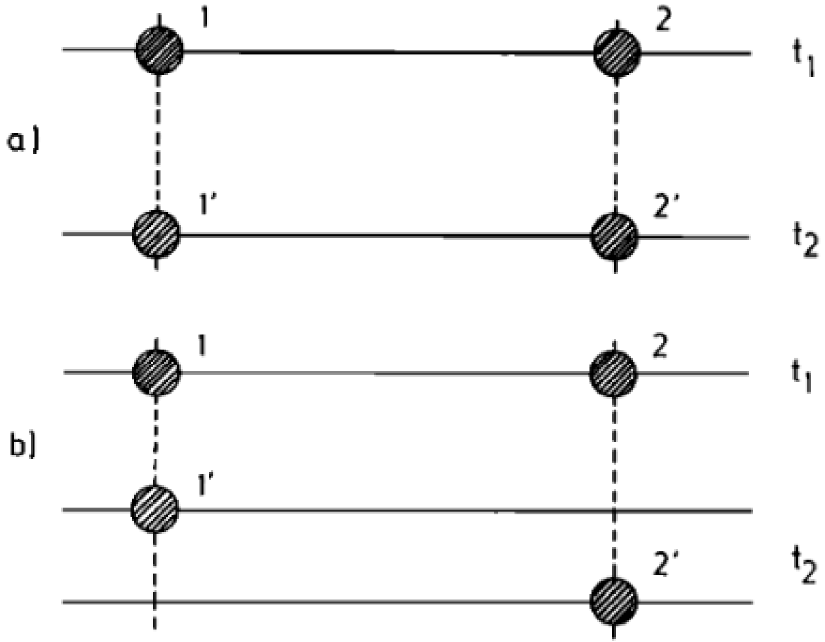


Parnell et al. 2010, ApJ



Aulanier et al. 2006, Solar Phys.

General magnetic reconnection (GMR)



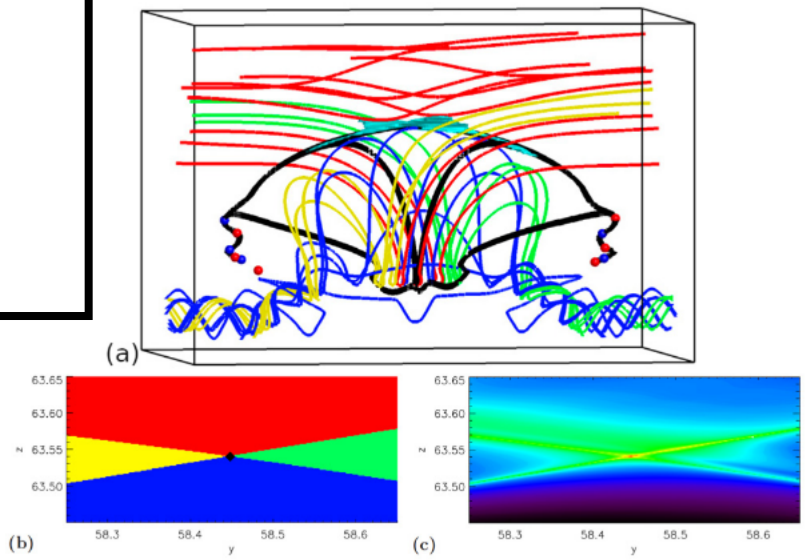
Schindler et al. 1988, JGR

$$\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B} = \boldsymbol{R} \quad \text{in} \quad D \subset \mathbb{R}^3$$

$$\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B} = \boldsymbol{0} \quad \text{in} \quad \mathbb{R}^3 \setminus D$$

Reconnection rate:

$$R_D = \max_D \int \boldsymbol{R} \cdot \boldsymbol{b} \, \mathrm{d}x$$



Parnell et al. 2010, ApJ

Eyink's critique (Eyink 2015, ApJ)

In turbulent plasmas, there are no discrete and isolated D.

In a turbulent inertial range, R can by tiny but $\nabla \times R$ need not be.

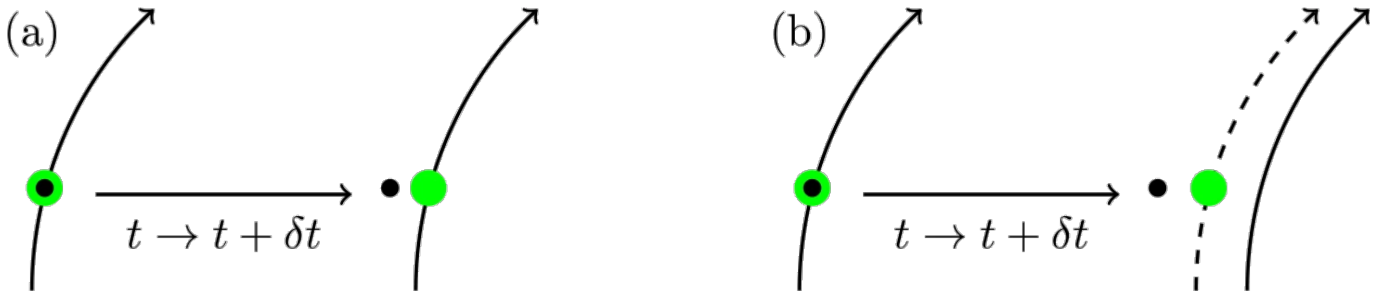
Magnetic reconnection is related to:

the slip velocity source

$$\boldsymbol{\Sigma} = -\frac{(\nabla \times \boldsymbol{R})_\perp}{|\boldsymbol{B}|}$$

This is the rate of velocity slip per unit arclength along a field line.

A simpler interpretation



$$\frac{\partial \boldsymbol{b}}{\partial t} = \frac{1}{|\boldsymbol{B}|}\left(\frac{\partial \boldsymbol{B}}{\partial t}\right)_{\perp}$$

Induction equation

$$\frac{\partial \boldsymbol{b}}{\partial t} = \frac{1}{|\boldsymbol{B}|}(\nabla \times (\boldsymbol{u} \times \boldsymbol{B}) - \nabla \times \boldsymbol{R})_{\perp} = \frac{1}{|\boldsymbol{B}|}(\nabla \times (\boldsymbol{u} \times \boldsymbol{B}))_{\perp} + \Sigma$$

Further connections

$$\boldsymbol{j} = \boldsymbol{B} \times \frac{\boldsymbol{F}}{|\boldsymbol{B}|^2} + \alpha \boldsymbol{B}, \quad \textbf{where} \quad \boldsymbol{F} = \boldsymbol{j} \times \boldsymbol{B}, \quad \alpha = \frac{(\nabla \times \boldsymbol{B}) \cdot \boldsymbol{B}}{|\boldsymbol{B}|^2}$$

Further connections

$$\boldsymbol{j} = \boldsymbol{B} \times \frac{\boldsymbol{F}}{|\boldsymbol{B}|^2} + \alpha \boldsymbol{B}, \quad \text{where} \quad \boldsymbol{F} = \boldsymbol{j} \times \boldsymbol{B}, \quad \alpha = \frac{(\nabla \times \boldsymbol{B}) \cdot \boldsymbol{B}}{|\boldsymbol{B}|^2}.$$

$$\boldsymbol{j} = \lambda \boldsymbol{B}_{f\perp} + \alpha \boldsymbol{B}, \quad \text{where} \quad \boldsymbol{B}_{f\perp} = \boldsymbol{B} \times \frac{\boldsymbol{F}}{|\boldsymbol{F}|}, \quad \lambda = \frac{(\nabla \times \boldsymbol{B}) \cdot \boldsymbol{B}_{f\perp}}{|\boldsymbol{B}|^2}.$$

Further connections

$$\boldsymbol{j} = \boldsymbol{B} \times \frac{\boldsymbol{F}}{|\boldsymbol{B}|^2} + \alpha \boldsymbol{B}, \quad \textbf{where} \quad \boldsymbol{F} = \boldsymbol{j} \times \boldsymbol{B}, \quad \alpha = \frac{(\nabla \times \boldsymbol{B}) \cdot \boldsymbol{B}}{|\boldsymbol{B}|^2}.$$

Field-aligned current

$$\boldsymbol{j} = \lambda \boldsymbol{B}_{f\perp} + \alpha \boldsymbol{B}, \quad \textbf{where} \quad \boldsymbol{B}_{f\perp} = \boldsymbol{B} \times \frac{\boldsymbol{F}}{|\boldsymbol{F}|}, \quad \lambda = \frac{(\nabla \times \boldsymbol{B}) \cdot \boldsymbol{B}_{f\perp}}{|\boldsymbol{B}|^2}.$$

Lorentz force

$$\boldsymbol{F} = \lambda \boldsymbol{B}_{f\perp} \times \boldsymbol{B} = \lambda |\boldsymbol{B}|^2 \frac{\boldsymbol{F}}{|\boldsymbol{F}|}.$$

For resistive MHD:

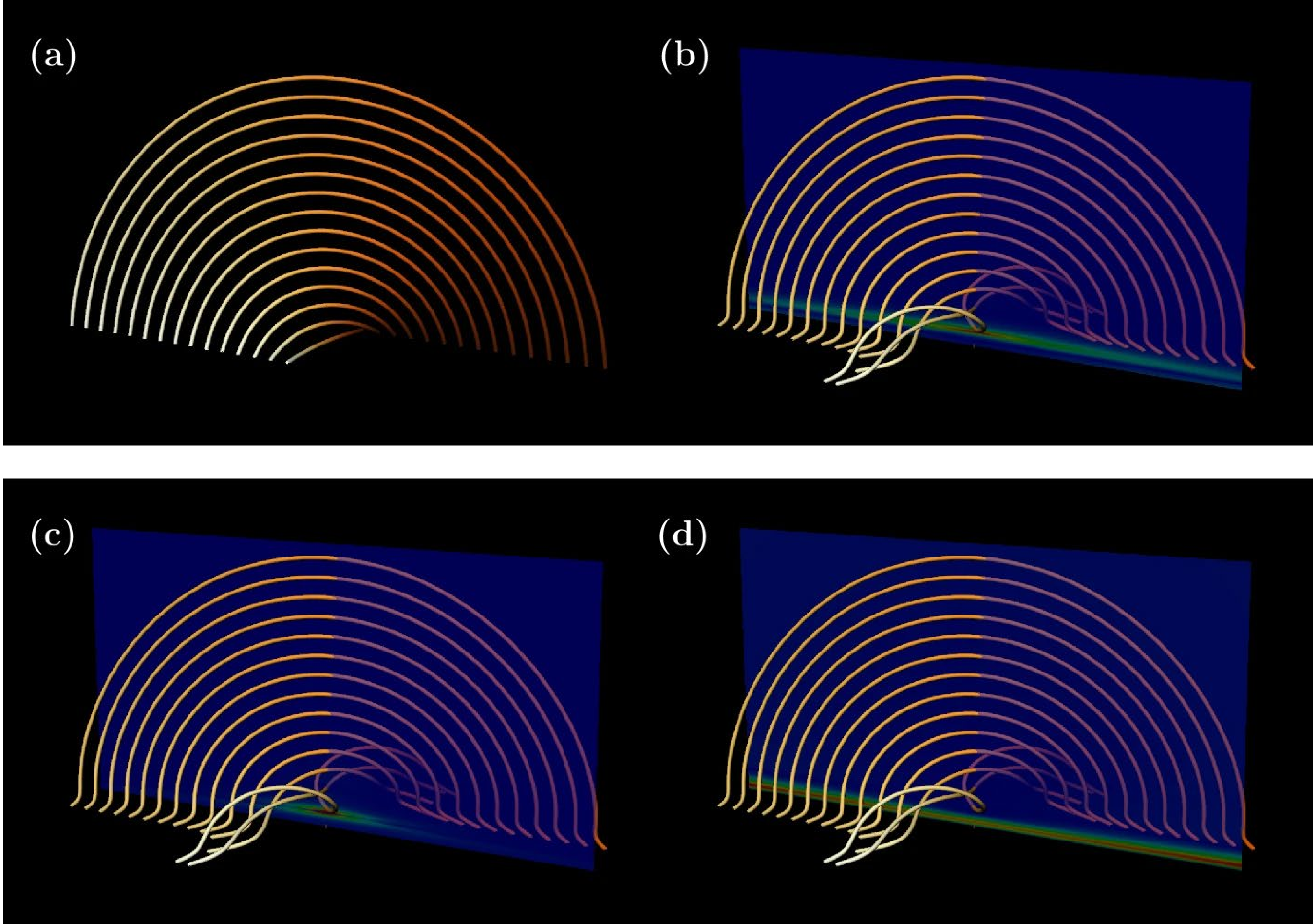
$$\Sigma = -\frac{1}{|\boldsymbol{B}|}(\boldsymbol{C}_1 + \eta\boldsymbol{C}_2),$$
$$\boldsymbol{C}_1 = \alpha\nabla\eta \times \boldsymbol{B} - \lambda(\nabla\eta \cdot \boldsymbol{B})\frac{\boldsymbol{F}}{|\boldsymbol{F}|},$$
$$\boldsymbol{C}_2 = (\lambda\omega_1 - \nabla\lambda \cdot \boldsymbol{B})\frac{\boldsymbol{F}}{|\boldsymbol{F}|} + \lambda(\alpha + \omega_2)\boldsymbol{B}_{f\perp} + \nabla\alpha \times \boldsymbol{B},$$

where

$$\omega_1 = \nabla \times \boldsymbol{B}_{f\perp} \cdot \frac{\boldsymbol{F}}{|\boldsymbol{F}|}, \quad \omega_2 = \frac{(\nabla \times \boldsymbol{B}_{f\perp}) \cdot \boldsymbol{B}_{f\perp}}{|\boldsymbol{B}|^2}.$$

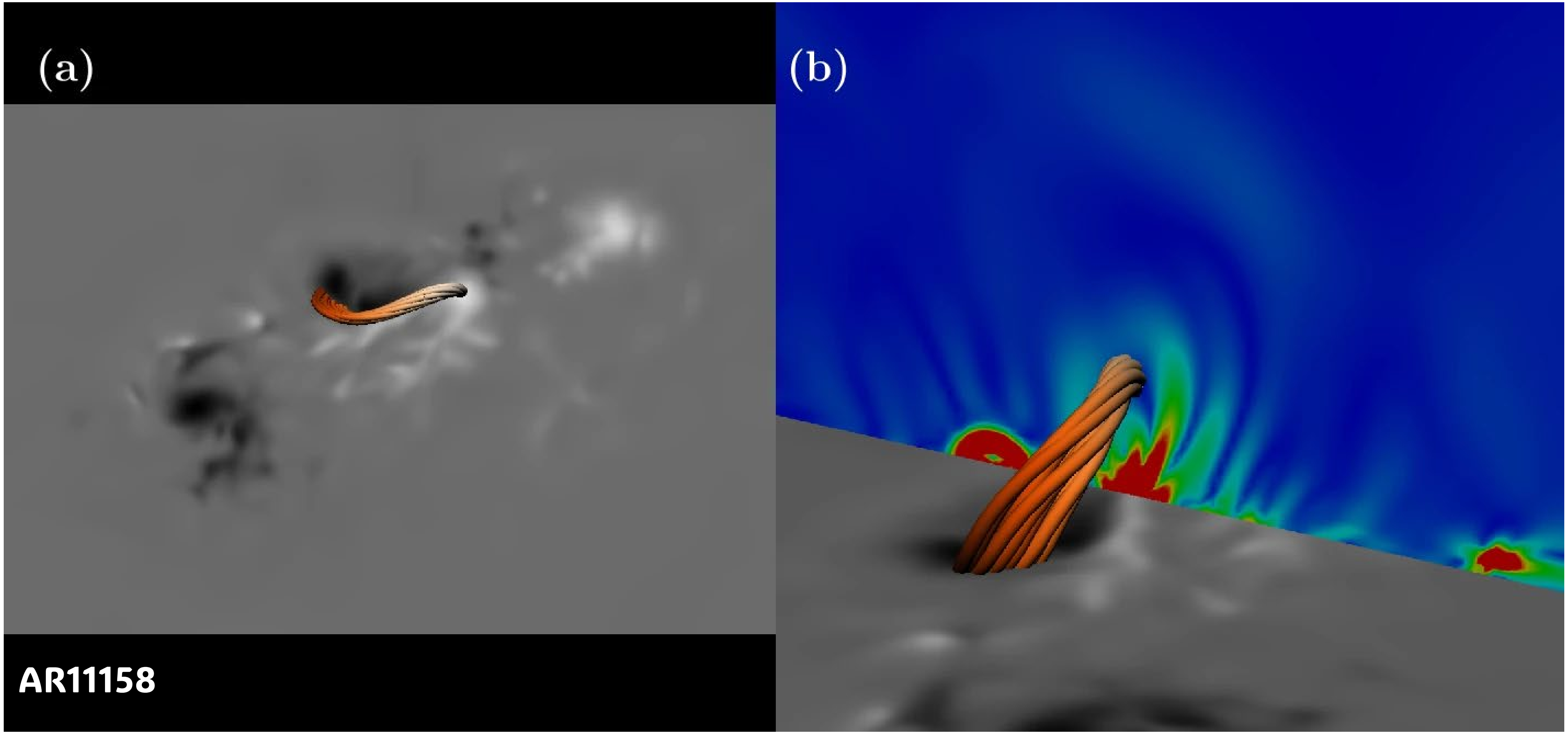
First moral **- the slippage rate links reconnection to important magnetic field elements: the Lorentz for and field-aligned currents**

Second moral **- all quantities are pointwise, i.e. they are local to small neighbourhoods**

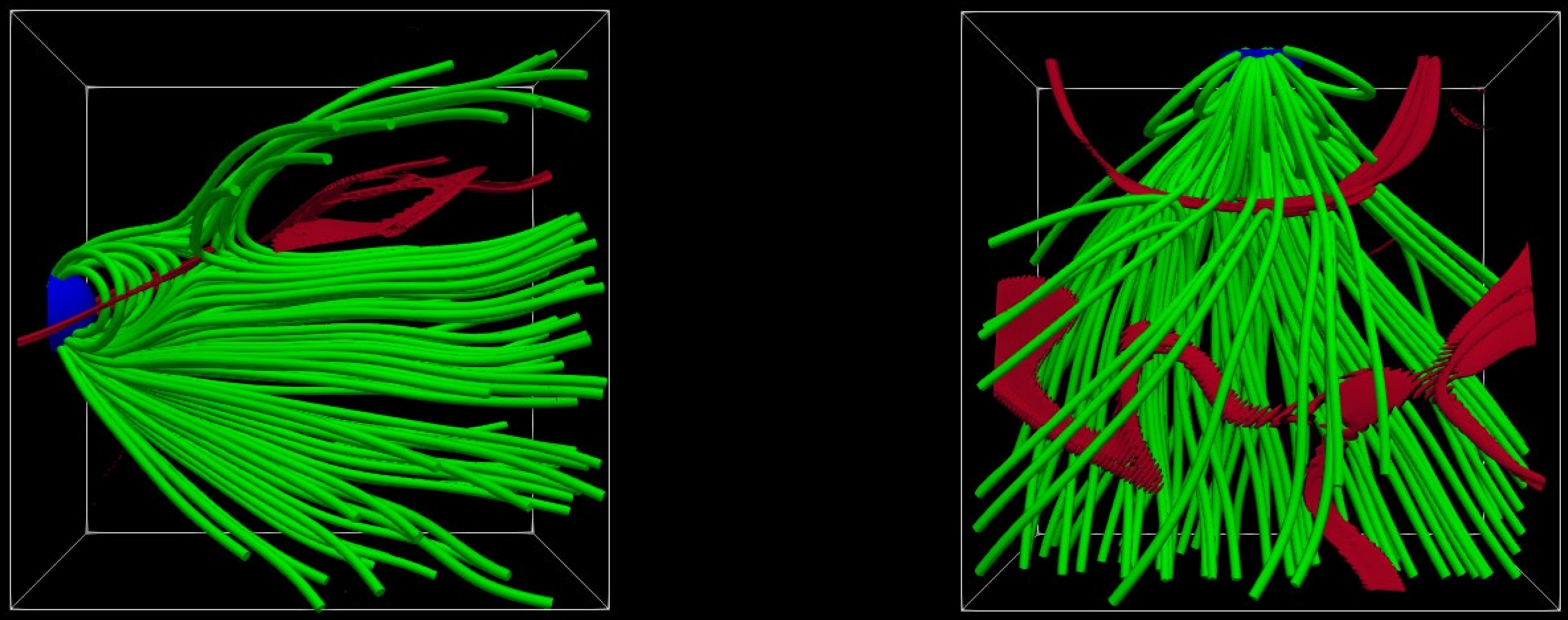

(a)
(b)
(c)
(d)

Force-free fields

$$\Sigma_{\rm NLFF} = -\eta \nabla \alpha \times \boldsymbol{b}.$$



Magnetosphere (nightside)



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Summary

- **A general description of reconnection (can describe turbulent and laminar plasmas)**
- **Connects reconnection to important magnetic properties (Lorentz force & FAC)**
- **It is a local theory (pointwise and easy to compute in simulations and extrapolations)**

For further details see

MacTaggart 2025, On field line slippage rates in the solar corona, *Solar Physics*, 300, 48