# Dynamical Galactic Halo Reconstruction from Rotation Curves in Self-Interacting Fuzzy Dark Matter

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Fuzzy Dark Matter (FDM) with an explicitly non-zero quartic self-interaction (SFDM) is shown to be a viable model for simultaneously fitting 17 dark-matter (DM)-dominated galaxies from the SPARC database, constraining both the boson mass, $m$, and the self-coupling constant, $g$, to values within the range $\log_{10} m\,[\mathrm{eV}/c^2] = \log_{10} 1.98 - 22^{+0.8}_{-0.6}$ and $\log_{10} g\,[\mathrm{Jm^3/kg}] = \log_{10} 1.45 - 28^{+0.4}_{-1.2}$; this is based on the combination of an appropriately constructed static super-Gaussian (SG) profile for the inner galactic core ('soliton') region, and a Navarro-Frenk-White (NFW) profile for the surrounding halo region. Identification of these parameters enables the explicit *dynamical* reconstruction of potential host halos for such galaxies, for which we outline a procedure with a proof-of-principle demonstration for two galaxies (UGCA444, UGC07866) shown to yield viable rotation curves over a dynamical period of $\mathcal{O}(1)$ Gyr.

## Gross-Pitaevskii-Poison equation

SFDM is described by the mass density $\rho(\mathbf{r},t) = |\Psi(\mathbf{r},t)|^2$of a complex scalar field $\Psi(\mathbf{r},t)$ obeying the GPPPE [1,2],

$$i\hbar\partial_t\psi(\mathbf{r},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + g\rho(\mathbf{r},t) + \Phi(\mathbf{r},t)\right]\psi(\mathbf{r},t) \qquad (1.1)$$

and

$$\nabla^2\Phi(\mathbf{r},t) = 4\pi G m[\rho(\mathbf{r},t) - \langle\rho(\mathbf{r},t)\rangle]. \qquad (1.2)$$

## Bimodal halo description

A signature of (S)FDM halos is the presence of a solitonic core sitting in the centre of the well-known NFW density distribution [3]. The empirical [3] and Thomas-Fermi [1] profiles can nicely describe the non-interacting FDM and strongly interacting SFDM. Here we propose the SG profile to self-consistently account the influence from self-interaction between two limits. The radial density profile of SFDM halos can be approached by the bimodal fitting,

$$\rho(r) = \begin{cases} \rho_0 e^{-\ln 2 (r/r_c)^{\vartheta}} & r \le r_t \\ \rho_h[(r/r_h)(1+r/r_h)^2]^{-1} & r > r_t \end{cases} \qquad (2)$$

where $r_c$, $r_t$ and $r_h$ are the core, transition radii and NFW length scale, respectively. The continuity of $\rho(r)$ is imposed at $r_t$ the for the value of $\rho_h$. The core is also found to be the ground-state of the GPPE, and Ref. [4] has found out that the key parameter to describe the core is the dimensionless interacting strength

$$\Gamma_g = g/g_* \quad \text{with} \quad g_* \approx 200\frac{\hbar^4}{GM_c^2m^3} \qquad (3)$$

and $\vartheta = \vartheta(\Gamma_g)$ and the following relations

$$r_c = \left[\frac{B_g\hbar^2}{4\pi G\rho_0 m^2}\left(1+\sqrt{1+15A_0\Gamma_g/A_g}\right)\right]^{1/4}, \qquad (4)$$

$$v_0 = \sqrt{\frac{GM_c}{r_c}} = \sqrt{\frac{4\pi G\rho_0 r_c^2}{\vartheta(\ln 2)^{3/\vartheta}}\Gamma\left(\frac{3}{\vartheta}\right)}, \qquad (5)$$

And

$$M_c = 4\pi\eta\rho_0 r_c^3 \qquad (6)$$

where $A_g = A_g(\Gamma_g) = \sigma^2/\nu\zeta \lesssim 2A_{g=0}$ and $B_g = B_g(\Gamma_g) \sim \mathcal{O}(1)$ with the $\Gamma_g$-dependent shape parameters, $\eta$, $\sigma$, $\nu$ and $\zeta$[4,6]. Noting that the core becomes to be stabilized dominantly by the self interaction when $\Gamma_g > 3$ and the TF regime is when $\Gamma_g \gg 1$.

## Rotation Curves and SG-NFW fit

For a spherically symmetric density profile, the corresponding rotation curve is

$$v(r) = \sqrt{\frac{GM(r)}{r}} \qquad (7)$$

where $M(r)$ is the mass contained within radius $r$.

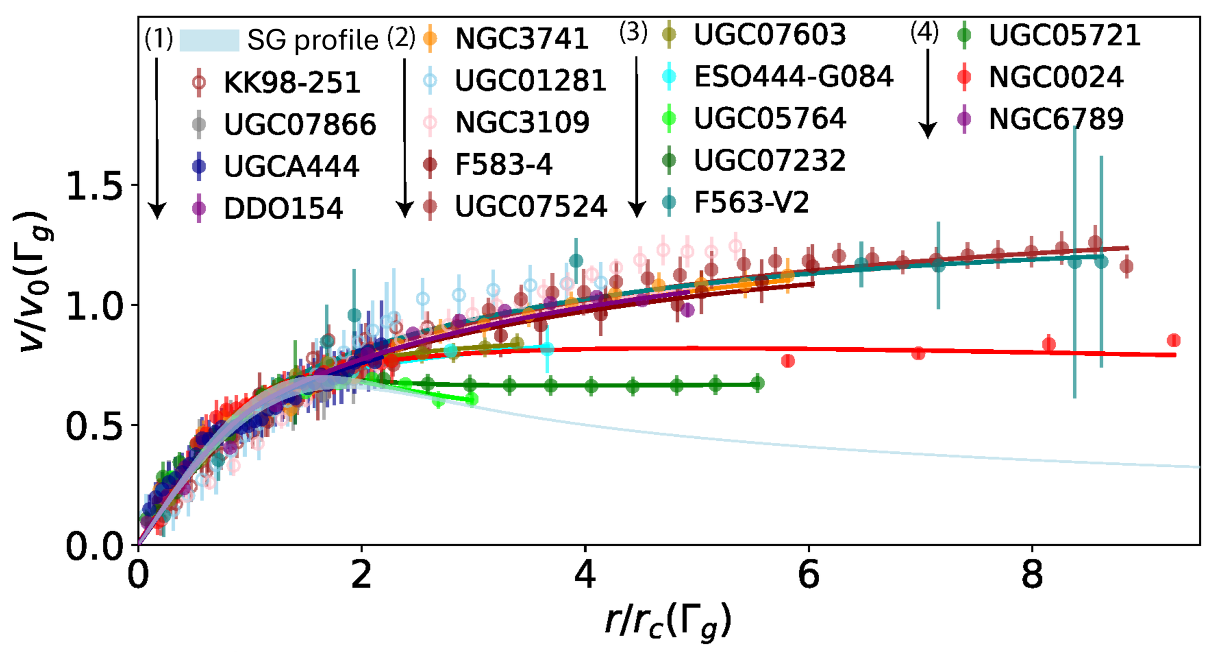


Figure 1: Scaled rotation curves by $v_0$ and $r_c$ according to Eqs (4) and (5) for the optimal common $m$ and $g$, describing the 17 selected DM dominated SPARC data [5].

We briefly address our fitting algorithm in Ref. [6]:

1. Considering a trial boson mass $m$,and $\Gamma_g$ (so $\vartheta$ is decided);
2. Guessing $\rho_0$ based on the first point in the SPARC rotation curve data point (see Fig. 1 ) to compute $r_c$ and $M_c$ according to Eqs. (4) and (6);
3. Conducting the bimodal fit according to Eq. (2) and mapping to rotation curve profile via Eq. (7);
4. Selecting fits via the $\chi^2$ analysis from the actual data;
5. Repeat step 2-4 with different $m$ and $\Gamma_g$.

According to Ref. [4], there is a $m - g$ degeneracy (see the dashed dotted line in Fig. 2), allowing us to extend the possible $m$ and $g$ pair, and Fig. 2 summarises the fit among the selected SPARC data in Fig. 1.

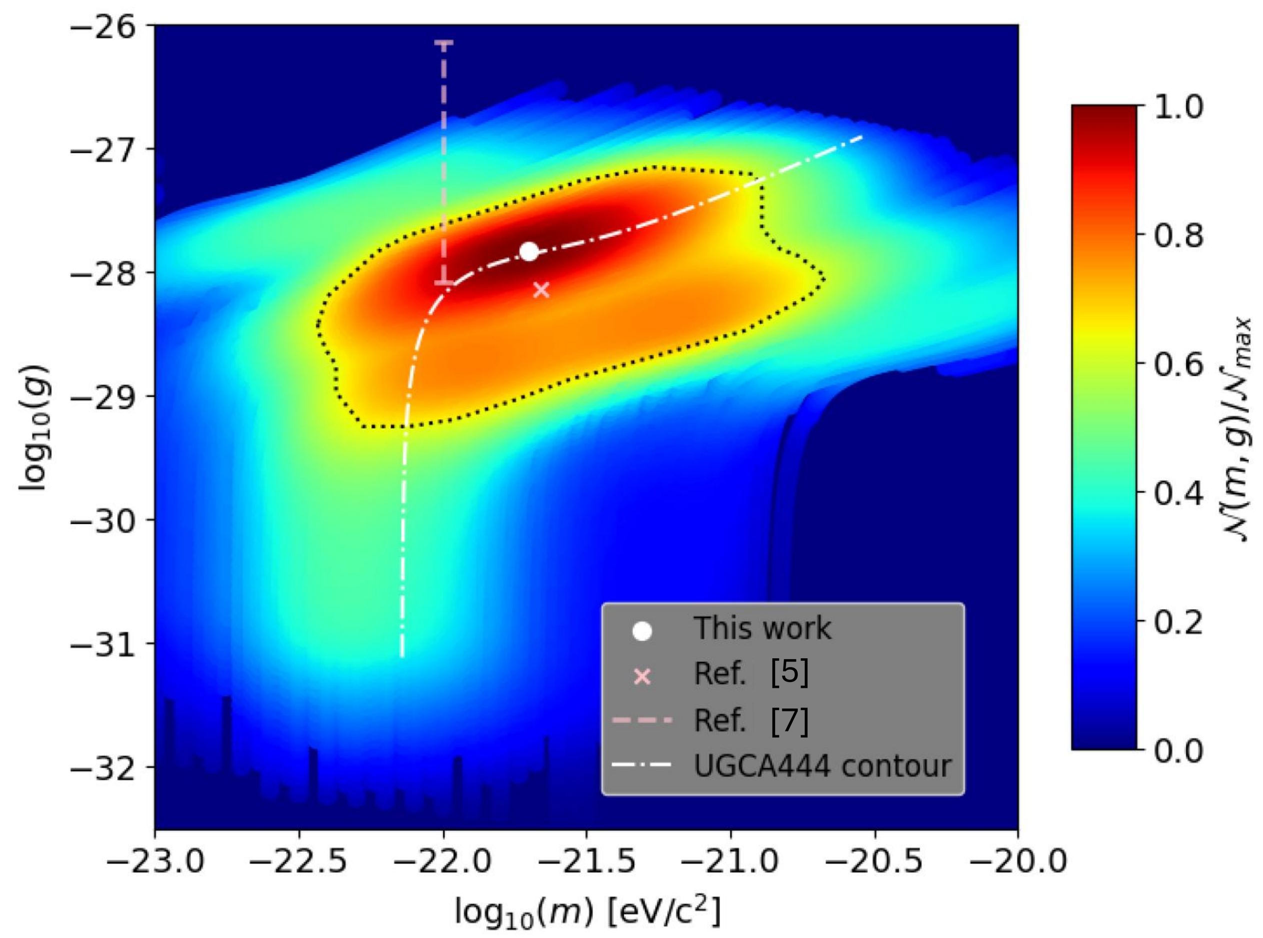


Figure 2: The density of soliton degeneracy curves in $(m, g)$ space according to the fitting algorithm for the selected 17 SPARC data. The optimized $(m, g)$ is passed the most many degeneracy curves.

## Merger simulation

Taking the optimized $m$ and $g$ and estimated total halo mass via Eq. (2), numerical halos, described by Eq. (1), can be generated from the collapse of many randomly distributing density lumps with a conserved total energy controlled by their initial configurations and widths. The semi-equilibrated halos can reproduce the rotation curves of UGCA444 and UGC07866.

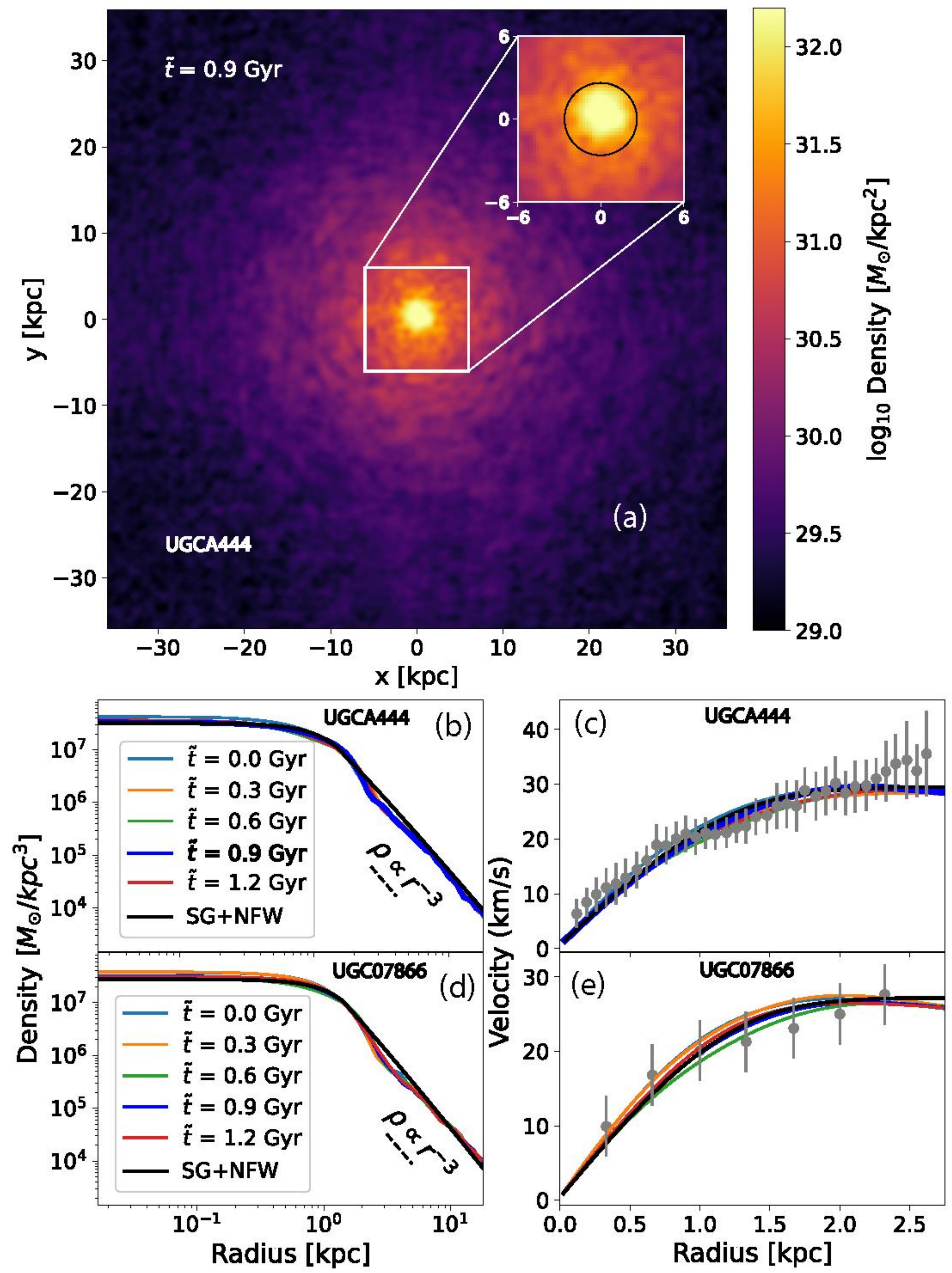


Figure 3: (a) Integrated density snapshot of the GPPE-simulated UGCA444 halo ($\Gamma_g = 16.6$). The inset is the close-up for the highlighted region with the black circle marking the spatial extent of the SPARC data. (b) and (c) show the corresponding radial profile for the density and rotation curve for UGCA 444. (d) and (e) are that for UGC07866 ($\Gamma_g = 12.9$).

## Remarks

For the first time, we found a consistent FDM boson mass to describe the rotation curves for the 17 DM-dominated galaxies in the SPARC data set by considering the self-interaction and a bimodal density profile. We demonstrate the first self-consistent GPPE simulations using the identified parameters to successfully reproduce the observation data. In addition, $\Gamma_g \in [4.8, 1630.8]$ are found for the selected galaxies, and therefore the core can be considered in the Thomas-Fermi regime.

## References

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