

MNRAS, 499, 2845 – 2883 (December 2020)

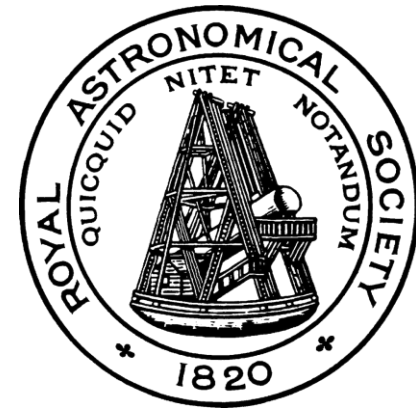
MNRAS, 527, 4388 – 4396 (January 2024)

MNRAS, 536, 3232 – 3241 (January 2025)

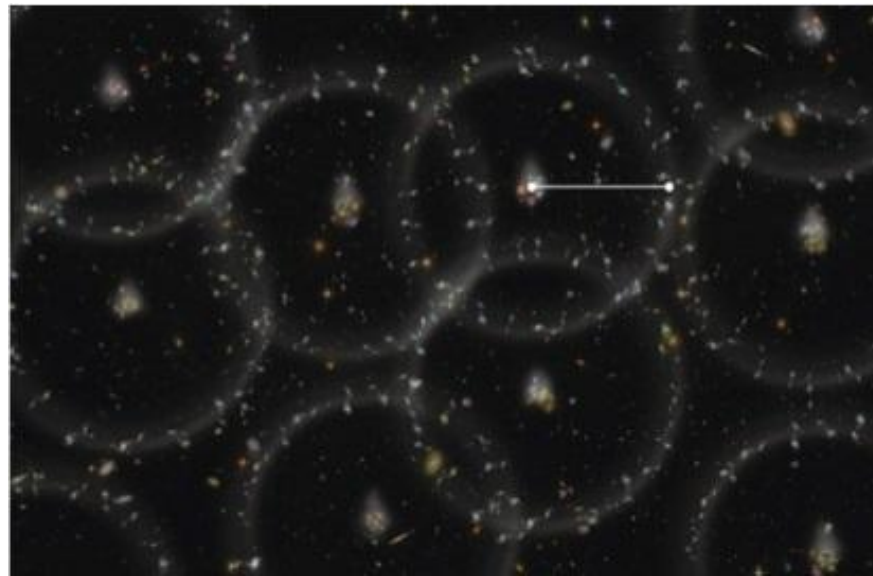
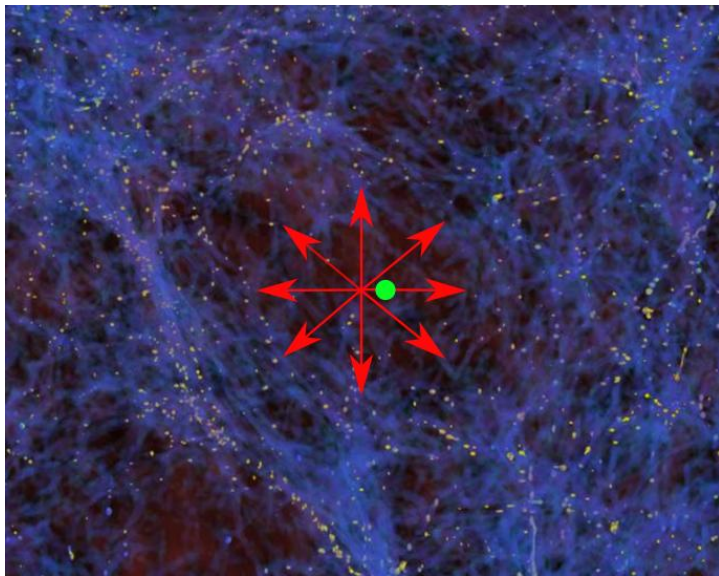
YouTube channel:
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The observed local supervoid could solve the Hubble and BAO tensions with Λ CDM



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Special Issue: "Theoretical and Observational Approaches to the Hubble Tension in Cosmology"

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Closing date will be near end of 2025 to allow time to include results presented here.



Guest Editor

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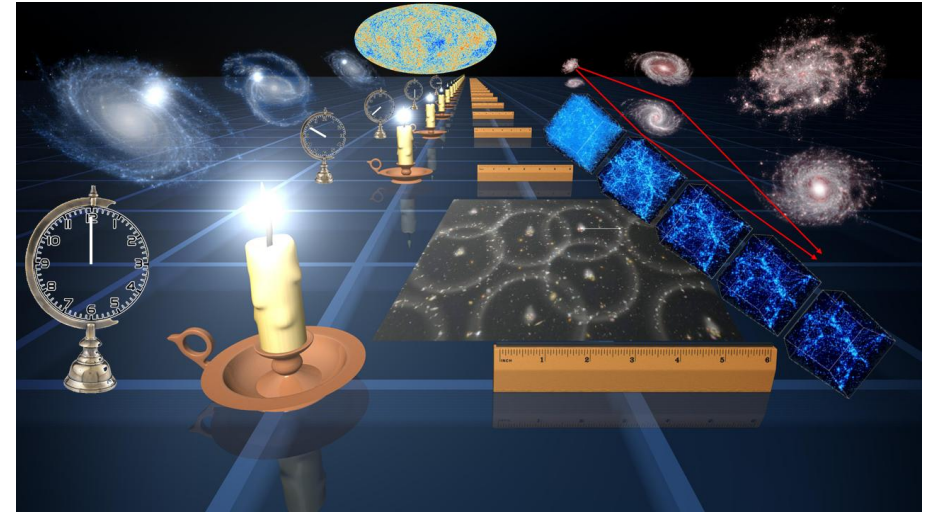


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Special Issue website:

https://www.mdpi.com/journal/galaxies/special_issues/BS2131Z18Q

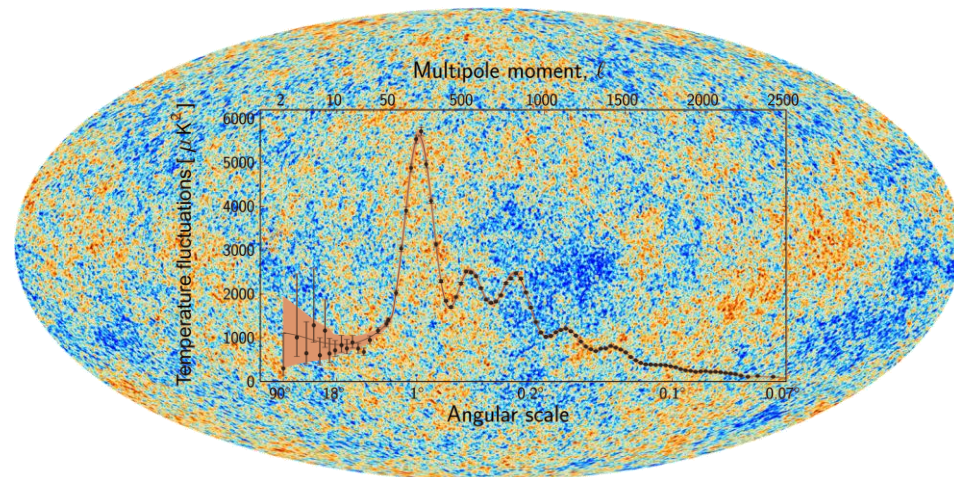
An important aim of this Special Issue is to exploit the increasing sensitivity of various techniques to give readers an idea of which ones actually show a tension with Λ CDM. This will give a deeper insight into the origin of the Hubble tension.



Two routes to the present expansion rate

- The currently popular Λ CDM model has achieved many successes (e.g. primordial D and He, galaxy cluster mass function at low z , cosmic shear)
- Its parameters are usually calibrated using CMB power spectrum (redshift $z = 1100$): unique shape
- Λ CDM then predicts the present expansion rate

$$H_0 \equiv \dot{a}/a$$



Credit: ESA (*Planck*)

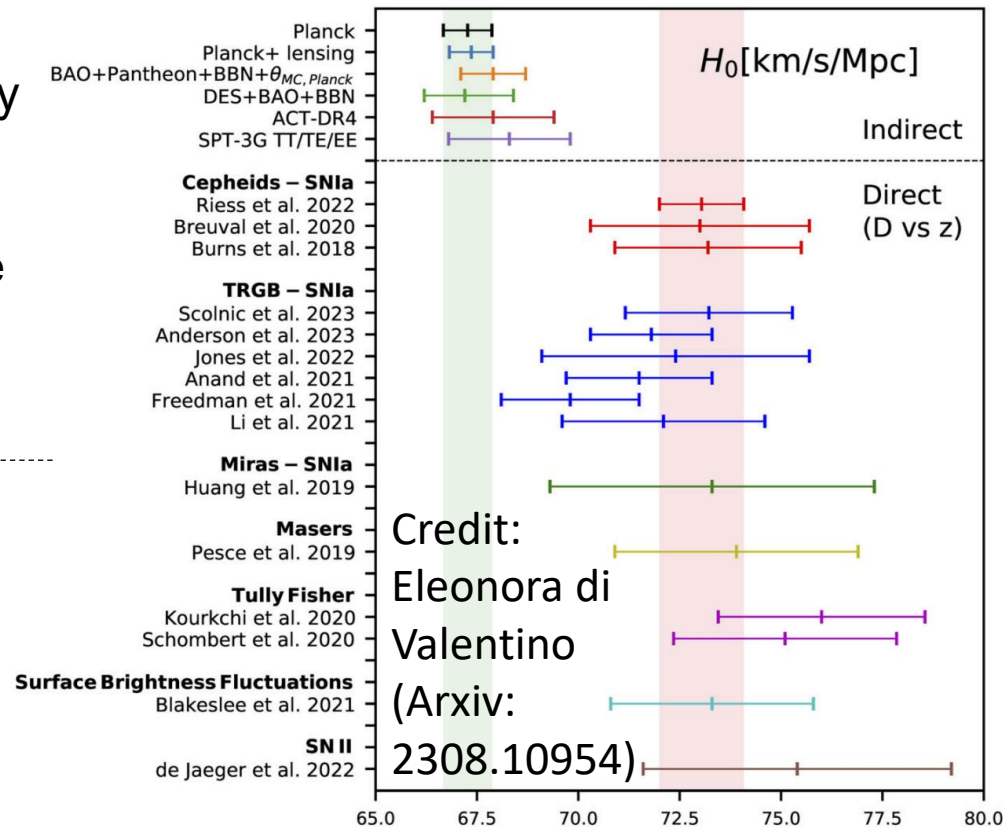
Two routes to the present expansion rate

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$$H_0 \equiv \dot{a}/a$$

- Local observations show a nearly linear relation between distance and redshift $z \equiv \lambda_{\text{obs}}/\lambda_{\text{emit}} - 1$
- If this is assigned entirely to the difference in the scale factor a at the time of emission and reception, we can infer

$$H_0 = \lim_{z \rightarrow 0} c \frac{dz}{dr}$$

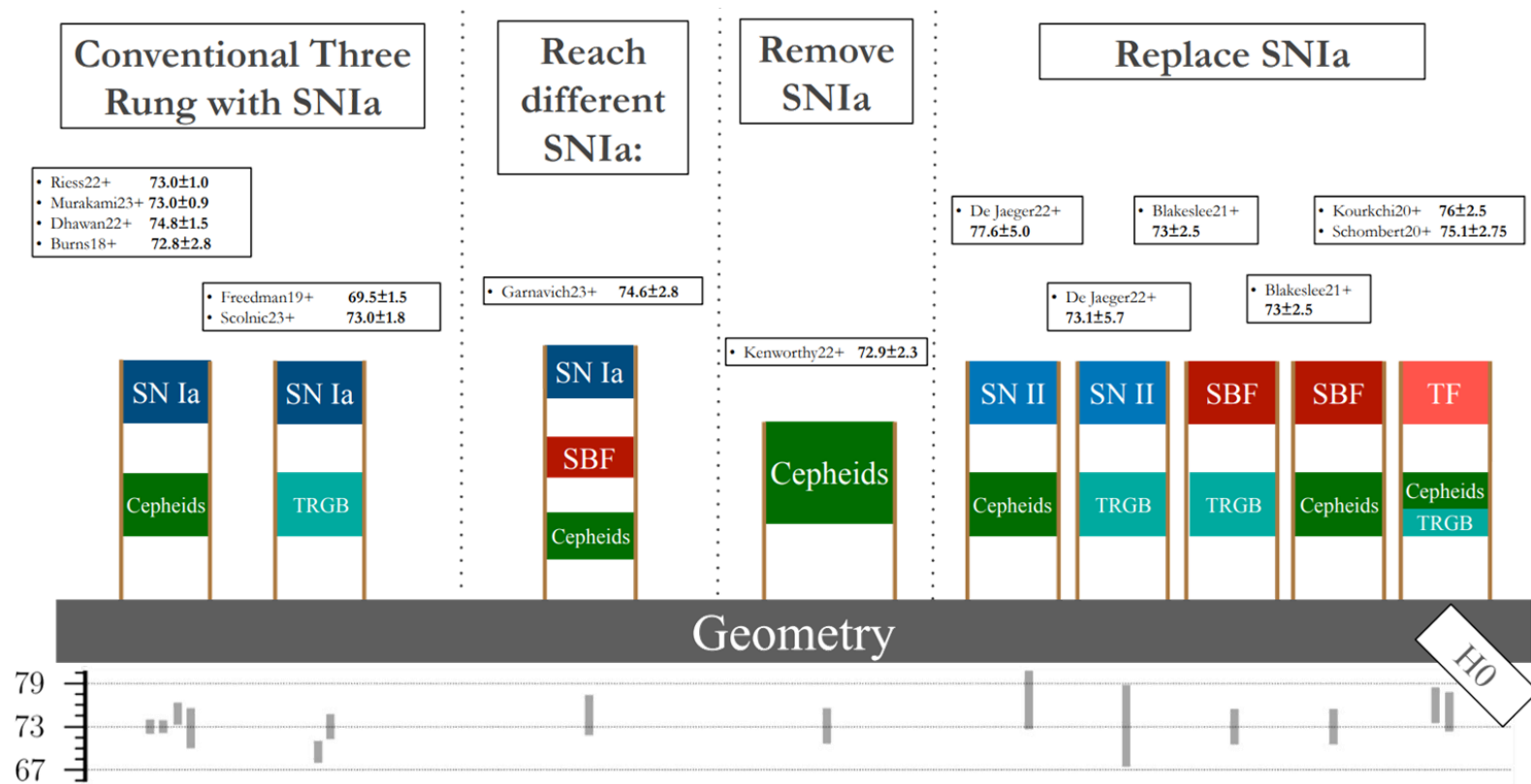


The local redshift gradient without SNe

- Can estimate without Cepheids in 1st rung and/or without Type Ia SNe in 2nd rung

- Megamasers give 73.9 ± 3.0 km/s/Mpc using only geometric distances out to 132 Mpc (Pesce+ 2020)

- HST Cepheids by Riess and JWST TRGB with Freedman give similar distances (Riess+ 2024).



Cosmic chronometers prefer *Planck* H_0

In this Letter, we use the latest results from the Dark Energy Spectroscopic Instrument (DESI) survey to measure the Hubble constant. Baryon acoustic oscillation (BAO) observations released by the DESI survey, allow us to determine H_0 from the first principles. Our method is purely data-driven and relies on unanchored luminosity distances reconstructed from Type Ia supernovae (SN Ia) data and $H(z)$ reconstruction from cosmic chronometers. Thus, it circumvents calibrations related to the value of the sound horizon size at the baryon drag epoch or intrinsic luminosity of SN Ia. We find $H_0 = 68.4_{-0.8}^{+1.0}$ km s⁻¹ Mpc⁻¹ at a 68% confidence level, which provides the Hubble constant at an accuracy of 1.3% with minimal assumptions. Our assessments of this fundamental cosmological quantity using the BAO data spanning the redshift range $z = 0.51$ – 2.33 agree very well with Planck's results and TRGB results within 1σ . This result is still in a 4.3σ tension with the results of the Supernova H_0 for the Equation of State.

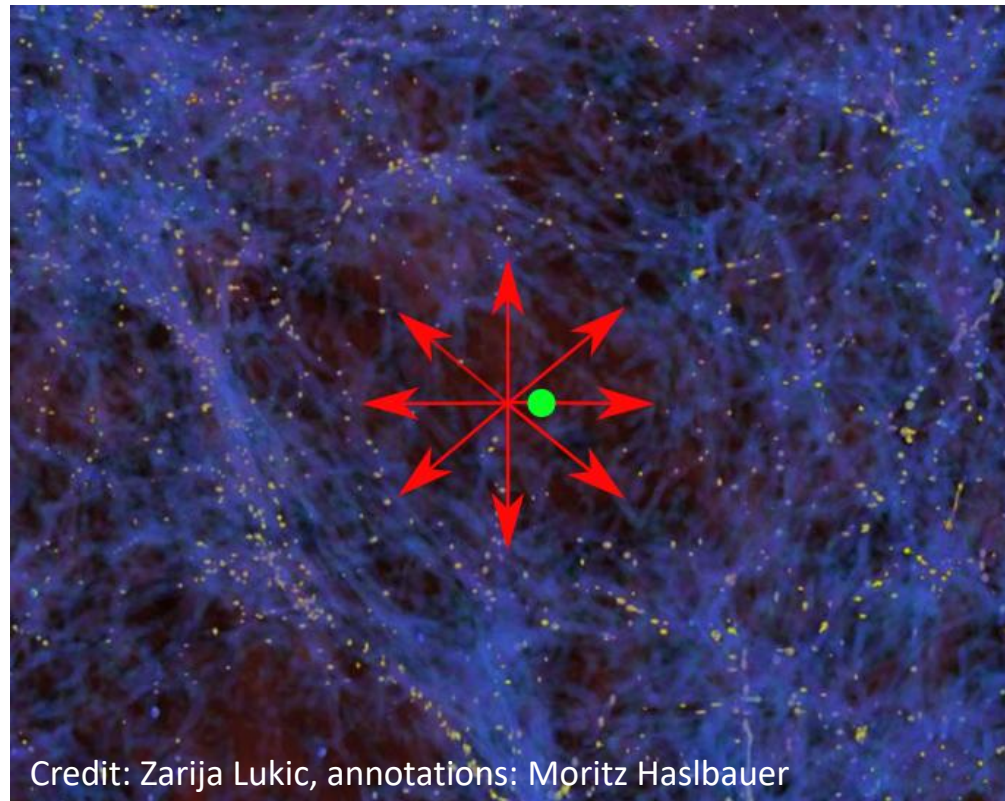
Guo+ 2025 (ApJL, 978, L33)

- Use uncalibrated SNe and BAOs only to constrain $a(t/t_U)$, which does not directly constrain H_0
- Use cosmic chronometers to constrain absolute timescale via dz/dt and thereby get H_0
- Data almost entirely at $z > 0.2$ as long timespans needed to get slope of time-redshift relation.

The impact of local structure

- The local measurement of H_0 is affected by peculiar velocities, which are velocities in the CMB rest frame (e.g., M31 is approaching us)
 - Impact of peculiar velocities should decrease as we consider more distant galaxies, which have more redshift from cosmic expansion
 - But if we are near the centre of a large void, outward peculiar velocity could rise at first and only start decreasing quite far out
- Peculiar velocities might skew the local H_0 .

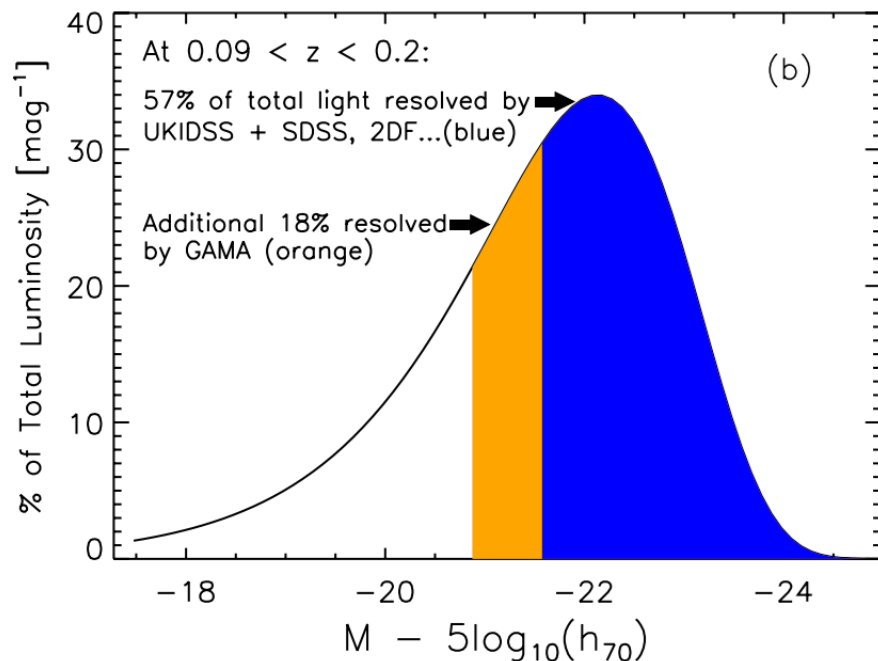
$$H_0 = \lim_{z \rightarrow 0 \text{ (but not too low)}} c \frac{dz}{dr}$$



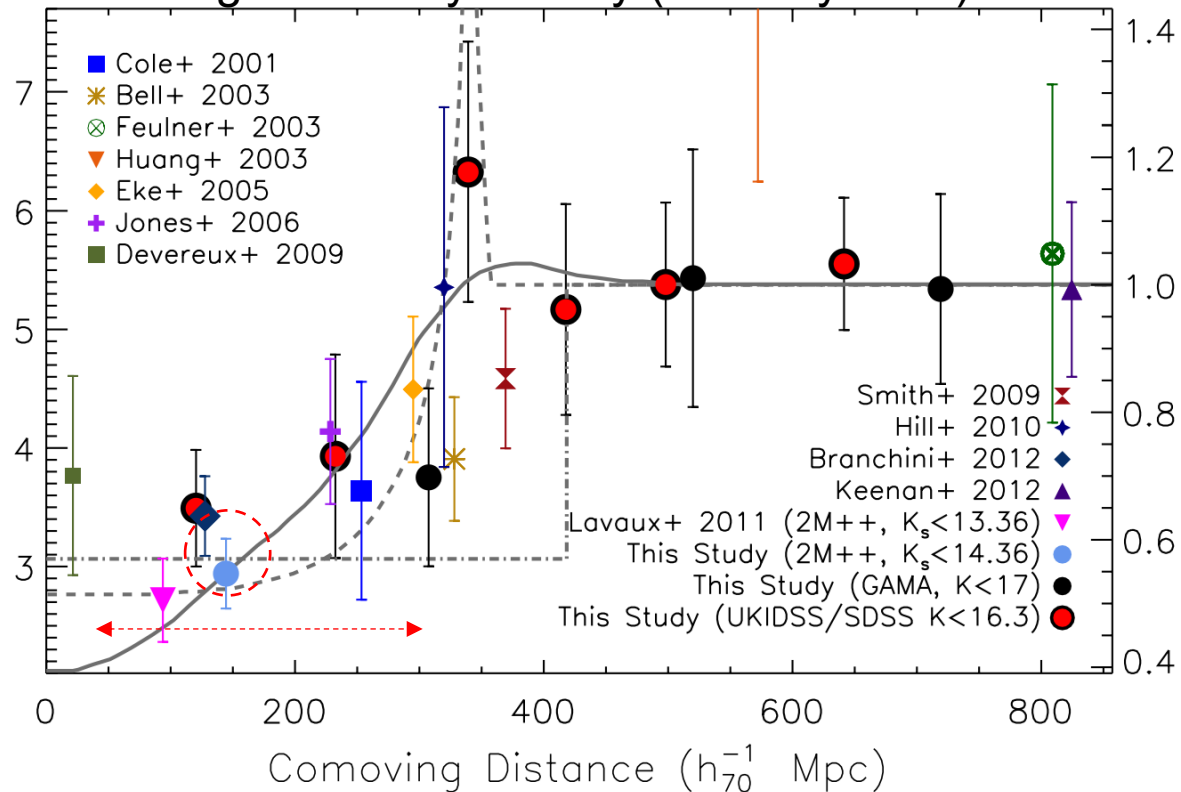
Credit: Zarija Lukic, annotations: Moritz Haslbauer

The KBC void (Keenan+ 2013)

- Galaxy number counts over 90% of sky and most of the luminosity function show an underdensity at 40 – 300 Mpc.

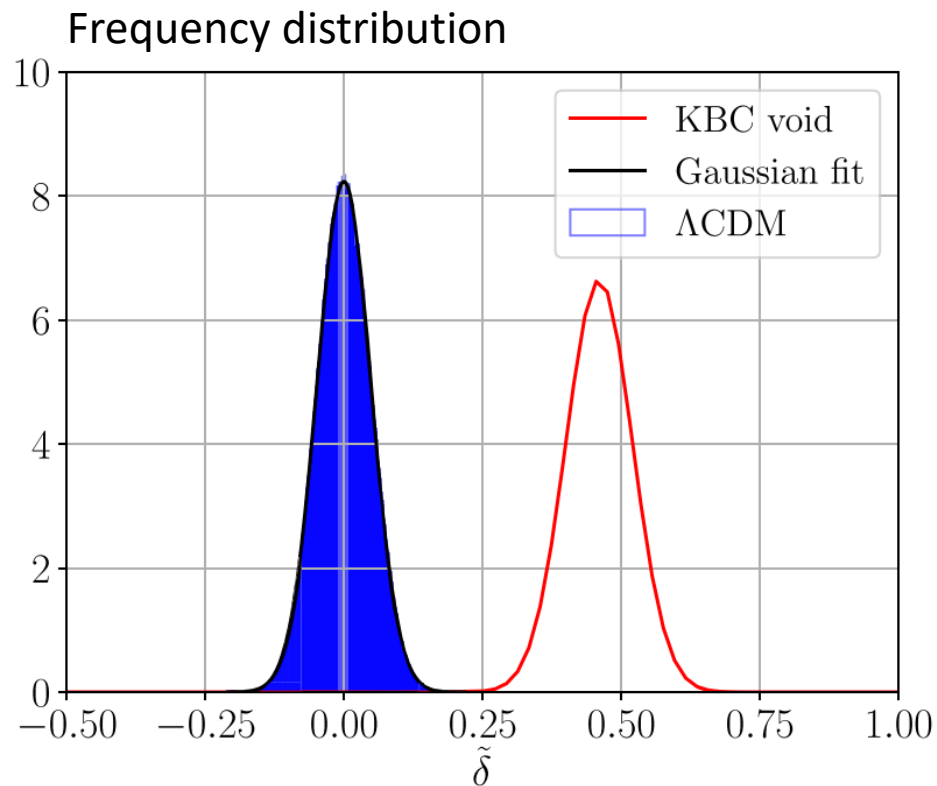


Comoving luminosity density (arbitrary units)



Analogues in Λ CDM

- Uniform grid of 10^6 vantage points in Millennium XXL (Angulo+ 2012), a Λ CDM N -body simulation with 4.1 cGpc box size.
 - Get total mass in halos with semi-analytic $M_* > 10^{10} M_\odot/h$ within 40 – 300 Mpc of each vantage point
 - Compare density to cosmic mean value
 - Enhance the density contrast 1.5x to allow for redshift space distortions: observers think they are seeing out to some distance d based on the redshift z , but outflows from local void mean the actual distance $< d$, reducing the galaxy number count
- Tension with Λ CDM is 6.04σ .



Increasing the cosmic variance

- Cosmic variance in local measurements of H_0 should be quite small in Λ CDM

➤ Cannot solve the Hubble tension

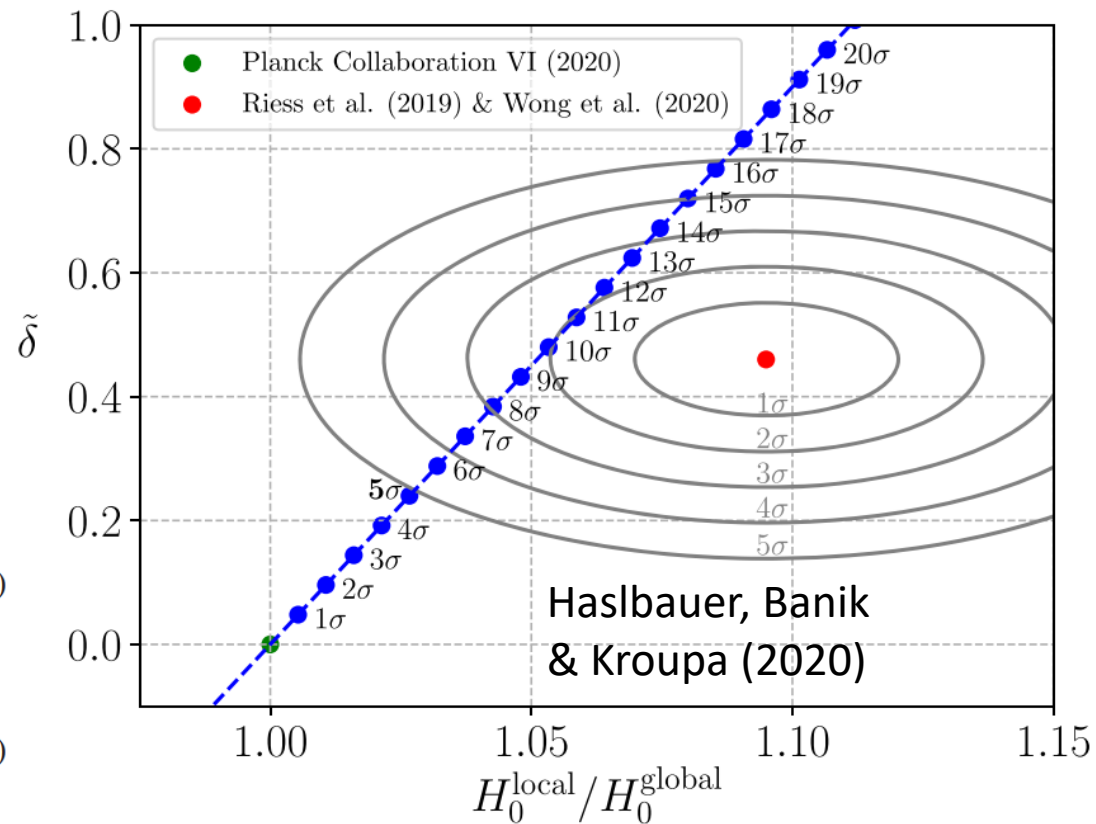
- But structure formation must be enhanced to explain the KBC void (local void also found by Wong+ 2022)

- Can we relate the observed local void to the Hubble tension?

$$\frac{\Delta H}{H} \equiv f\delta, \quad (18)$$

where e.g. Marra et al. (2013) showed that for $\delta \ll 1$ in Λ CDM,

$$f = \frac{\Omega_m^{0.6}}{3b}, \quad (19)$$

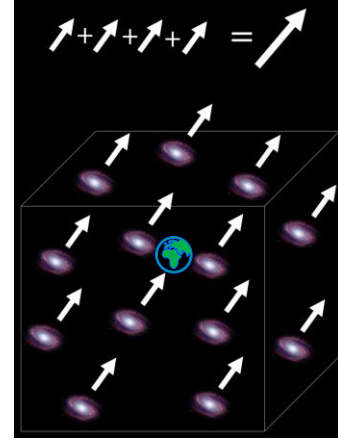


Bulk flow definition

- Ideally, we want average of 3D peculiar velocity vectors within spherical region
- Need to make do with line of sight (LOS) peculiar velocity of each galaxy
- Consider this as a vector pointing along the LOS
- Take average of these LOS peculiar velocity vectors, give higher weights to galaxies in more sparsely sampled regions (want to sample the velocity field)
- Galaxies weighted by $1/r^2$ (Peery+ 2018) & weighted mean found
- Final result tripled (Nusser 2016).

❖ Consequences:

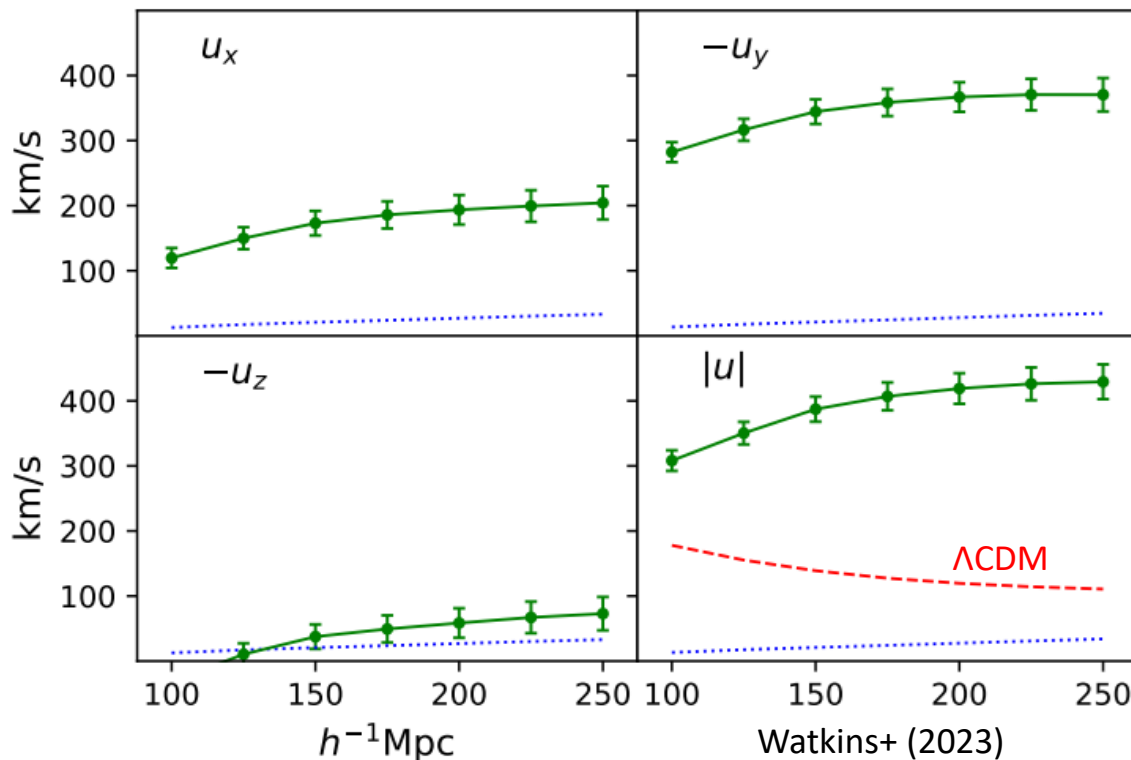
- If all galaxies have the same peculiar velocity $\mathbf{v}$, then $\mathbf{v}_{\text{bulk}} = \mathbf{v}$ despite projection effects
- Assumed H_0 has no effect on $\mathbf{v}_{\text{bulk}}$ as it affects peculiar velocities in spherically symmetric manner, which does not affect the vector average.



Credit: Abbe Whitford
(from AstroBites)

Bulk flow observations

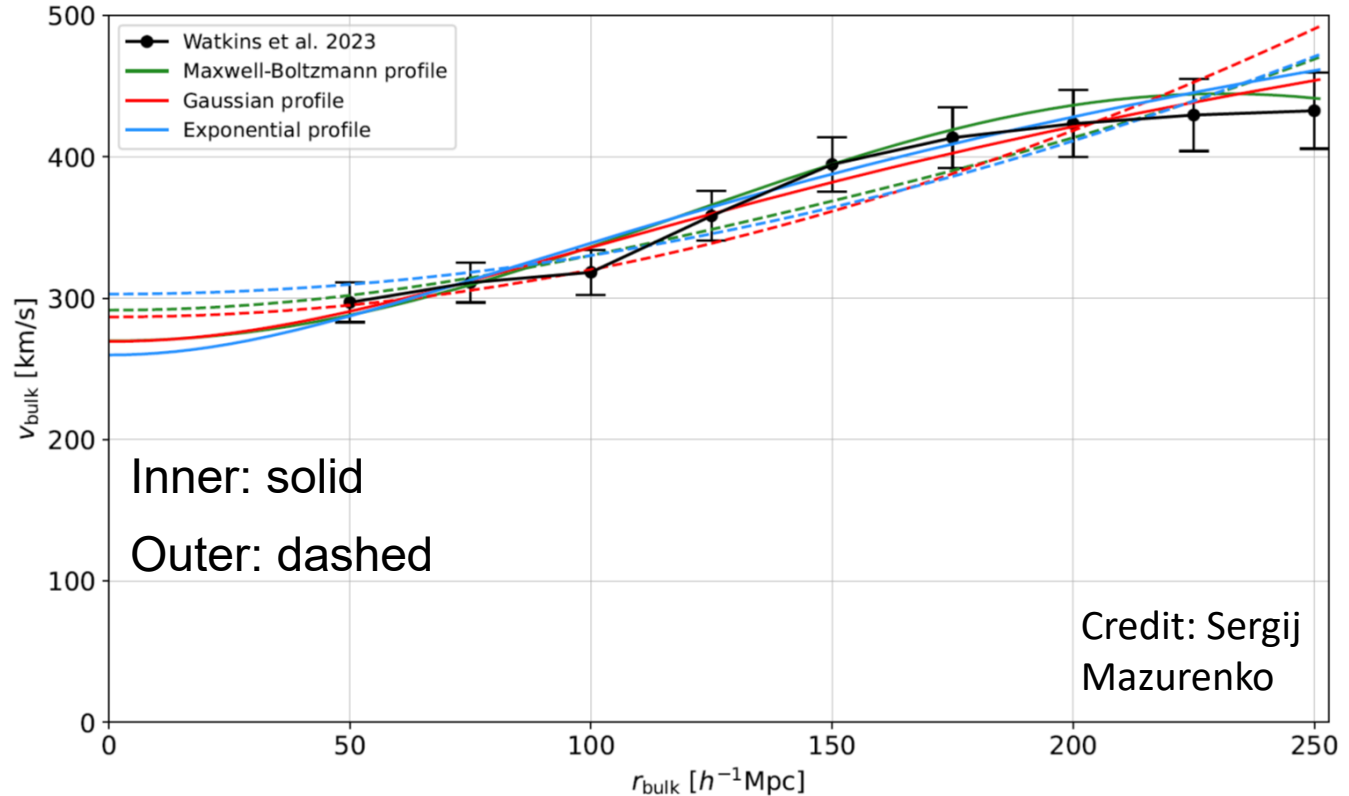
- Need galaxy survey with redshift-independent distances
 - Use CosmicFlows-4 (Tully+ 2023)
- Bulk flow analysis by Watkins+ (2023)
- Subsequent study by independent team (Whitford+ 2023) reports “excellent agreement” out to 173/ h Mpc, but their method did not extend further out
- Changing H_0 affects peculiar velocities in a spherically symmetric manner, not affecting vector average $\equiv$ bulk flow
 - Bulk flow tension $\neq$ Hubble tension.



Best-fit results with each profile

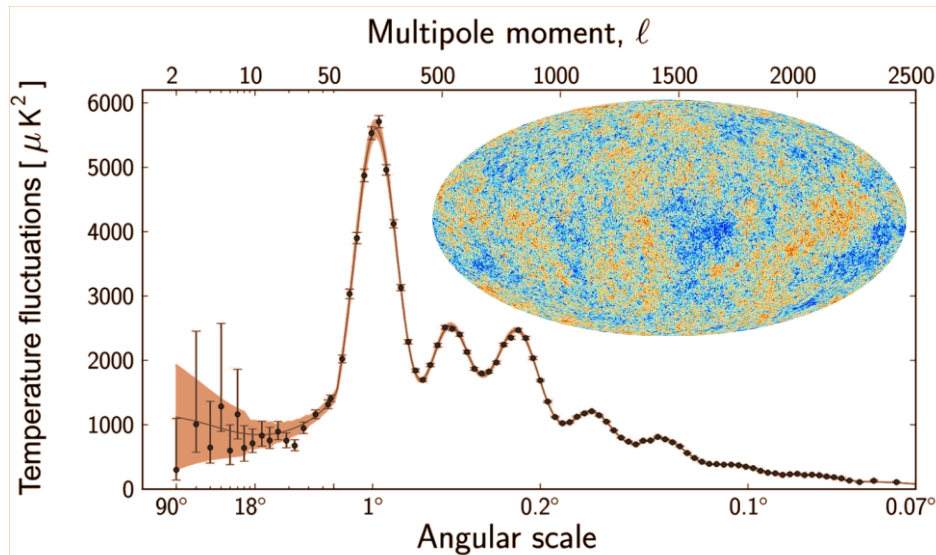
- Overall tension as equivalent no. of standard deviations for a single Gaussian

	Offset (Mpc)	Tension (σ)
Exponential	40	0.285
	55	1.089
Gaussian	40	0.132
	500	1.520
Maxwell-Boltzmann	210	0.116
	300	0.642

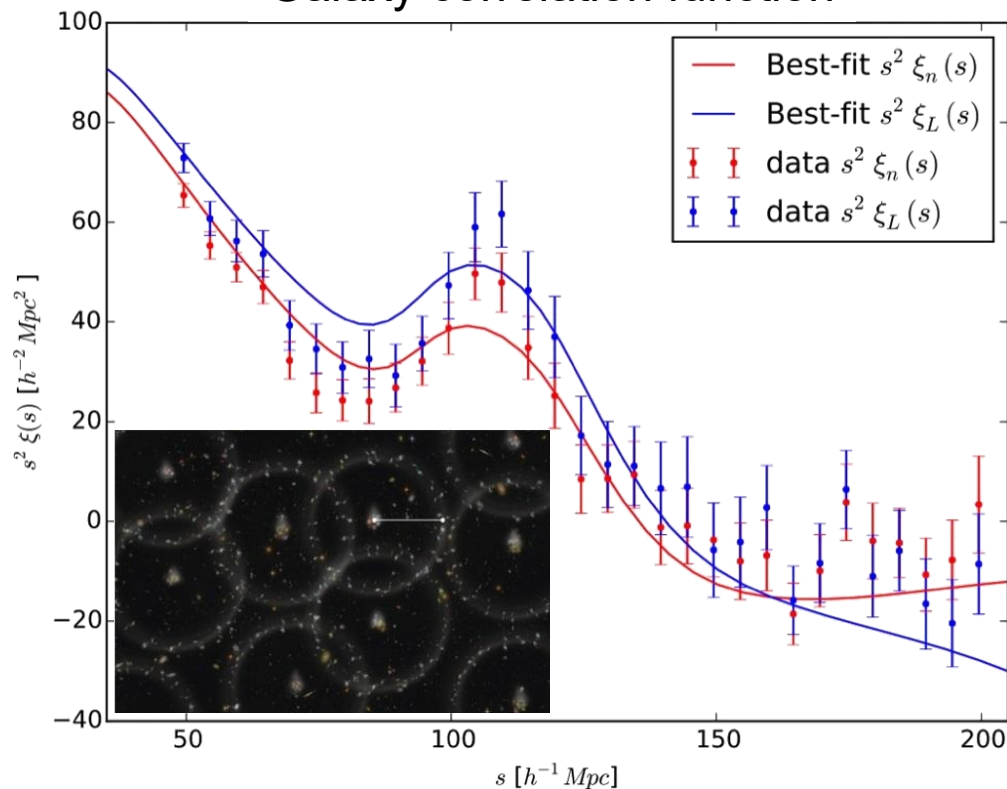


Baryon acoustic oscillation (BAO) basics

- Sound horizon at recombination frozen in place and imprinted on large-scale galaxy distribution
- Comoving length scale of first acoustic peak at the time of recombination is visible at late times!



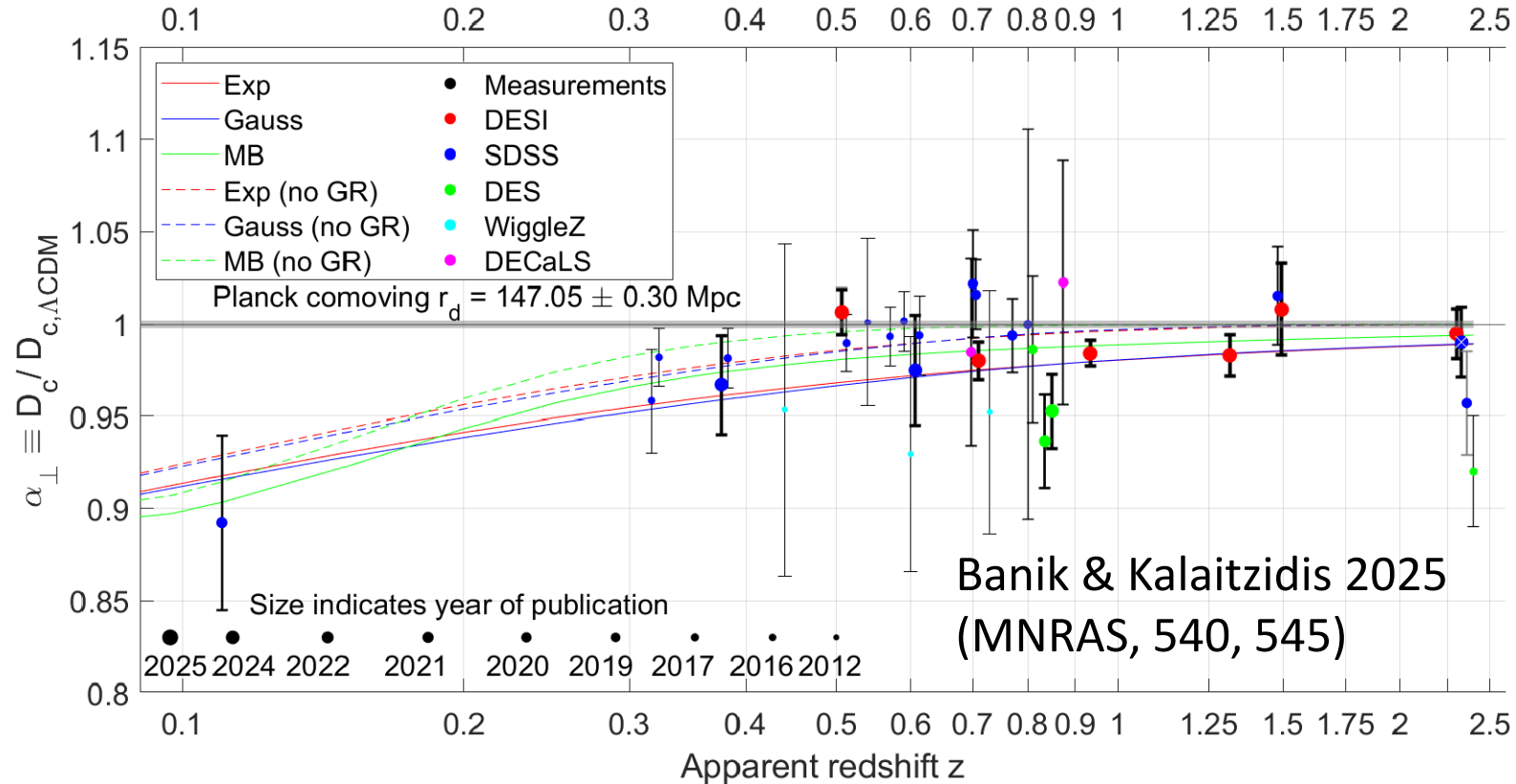
Galaxy correlation function



BAO results: angular scale

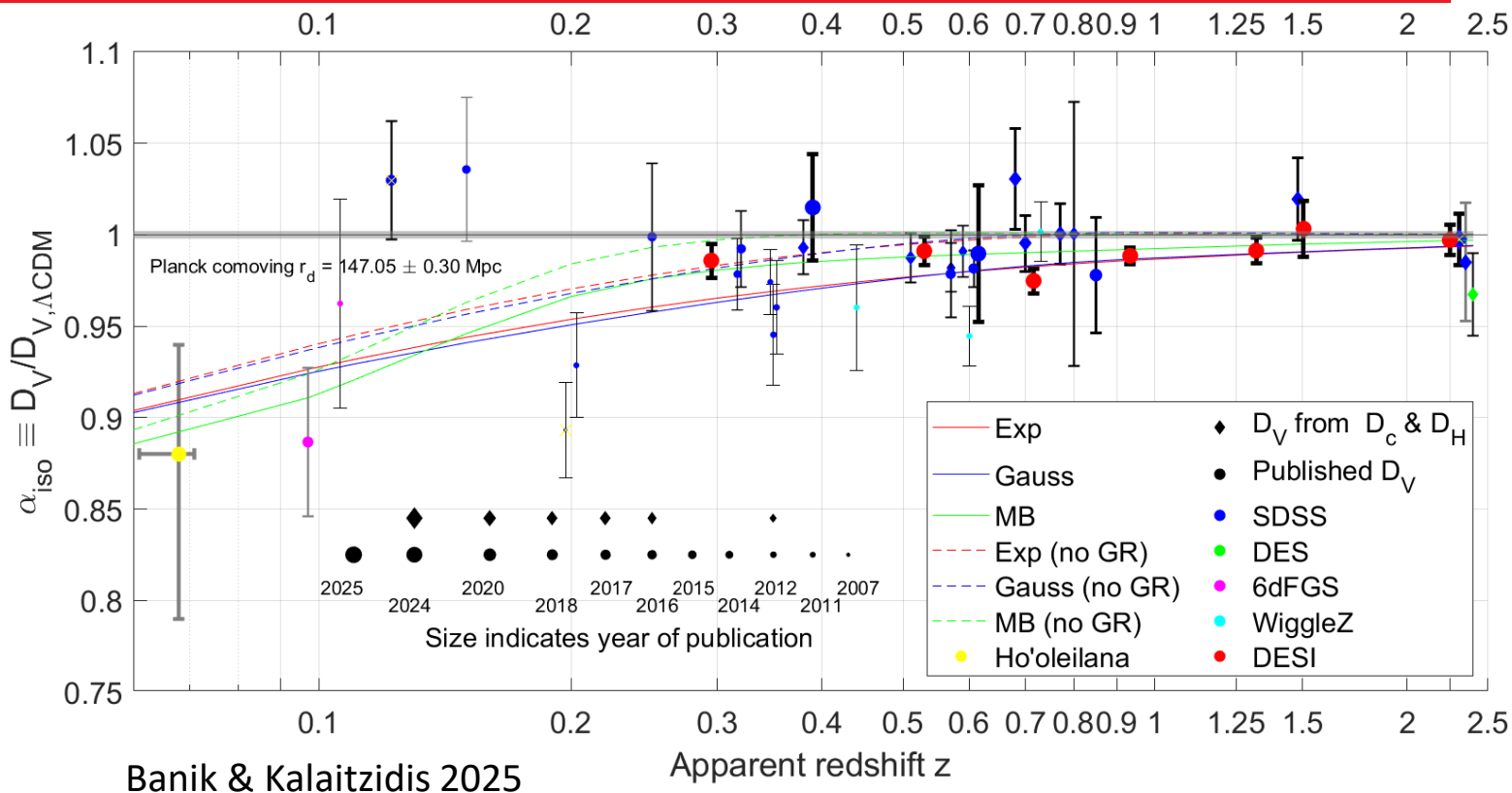
 D_c r_d

- BAO angular scale is ratio of comoving distance D_c to comoving length r_d of BAO ruler
- Take r_d from Planck Collaboration 2020
- Divide angles by expected value assuming *Planck* cosmology with no local void.



BAO results: isotropic average

- BAO is a 3D feature, so can combine angular scale with redshift depth to get the isotropic average:
 $D_V^3 \equiv z D_c^2 D_H$
- Averaging independent measurements (from orthogonal directions) reduces the uncertainty
- Many papers only report D_V as it is easier to obtain, esp. at low z .



Overall goodness of fit

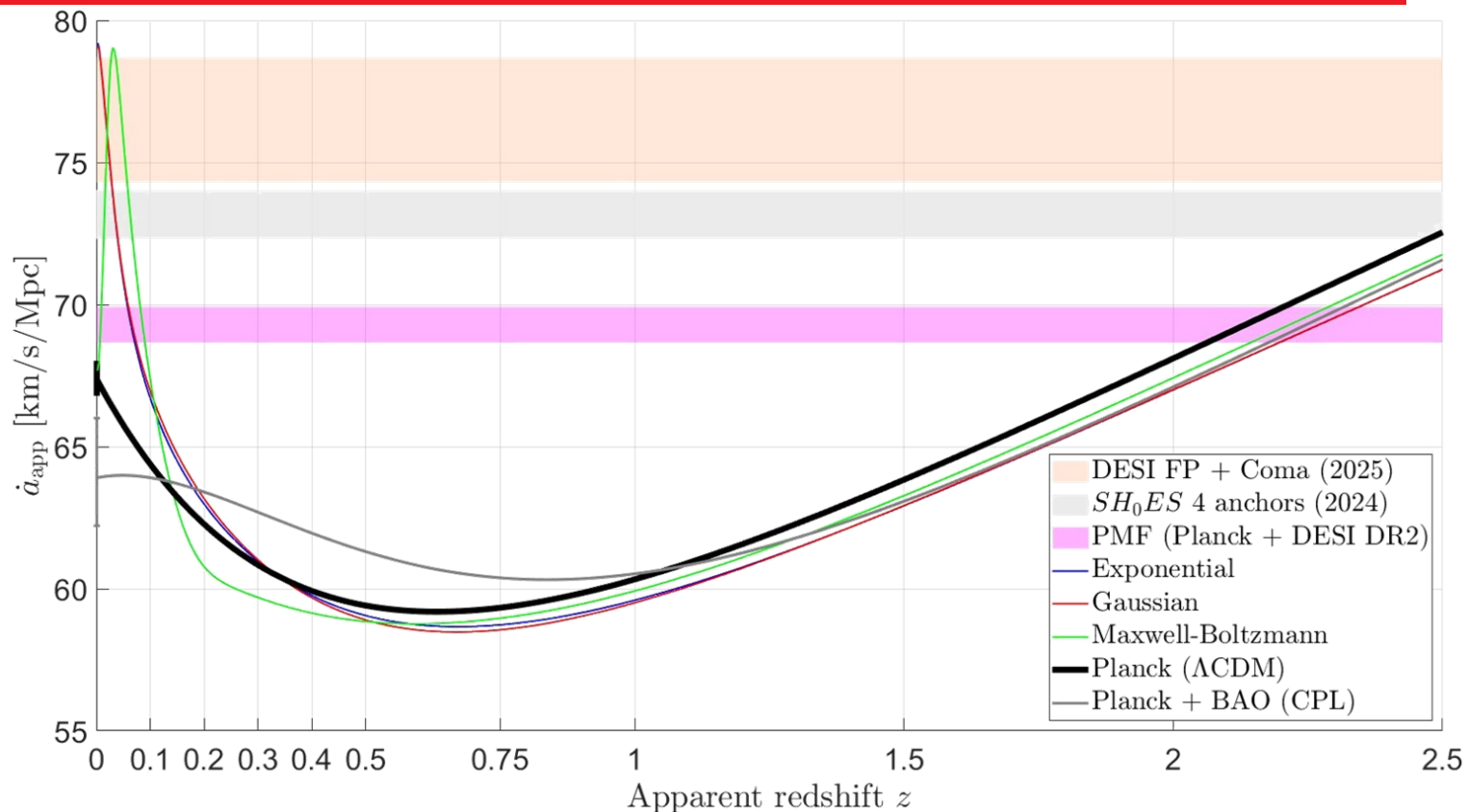
- BAO data not used to constrain model parameters in all cases.
- Most discriminative test was expected to be D_V due to availability of more points at low z and higher accuracy by averaging angular scale with redshift depth
- Clear preference for void models, which reduce χ^2 by about 24–28 (36–38 including D_c only points)
- Void models with 2020 parameters are vastly more likely than the homogeneous Planck cosmology.

Void profile	BAO observable used to calculate χ^2				α_{iso} & only $\alpha_{\perp}$
	$\alpha_{\perp} (D_c)$	$\alpha_{\parallel} (D_H)$	α_{AP}	$\alpha_{\text{iso}} (D_V)$	
Λ CDM	47.14	36.91	50.56	75.75	93.03
(no void)	(1.64 σ)	(1.44 σ)	(2.66 σ)	(3.27 σ)	(3.79 σ)
Exponential	46.07	34.96	50.15	50.31	55.85
	(1.55 σ)	(1.26 σ)	(2.62 σ)	(1.35 σ)	(1.19 σ)
Gaussian	48.74	36.34	52.72	51.25	56.76
	(1.77 σ)	(1.39 σ)	(2.84 σ)	(1.42 σ)	(1.26 σ)
Maxwell-Boltzmann	31.05	35.90	48.22	47.25	54.74
	(0.38 σ)	(1.35 σ)	(2.46 σ)	(1.11 σ)	(1.11 σ)
Data points	36	29	29	42	49

Note: 7 studies only report D_c and thus do not constrain D_V . These are included in final column.

Predictions for cosmic chronometers

- Assuming redshift is purely cosmological, get $a_{\text{app}} \equiv (1 + z)^{-1}$
- Can get dz/dt using cosmic chronometers, which are passively evolving galaxies
- Upper limit to stellar mass drops with time as more massive stars use up fuel faster
- No cosmological assumptions.



Conclusions

- Galaxy number counts show that we are in a significant underdensity out to ≈ 300 Mpc (Keenan+ 2013, Wong+ 2022) – good evidence across the whole electromagnetic spectrum (see introduction to Haslbauer+ 2020); significant tension with Λ CDM
- Given the nearly uniform initial conditions in the CMB, this requires significant local outflow
- Locally measured H_0 is indeed larger than predicted in Λ CDM by $\approx 10\%$ (di Valentino+ 2025)
 - Hubble & void tensions were linked in semi-analytic void model of Haslbauer+ 2020
- BAO observables deviate from *Planck* Λ CDM as expected in void model, which reduces χ^2 by 36–38 and thus tension (49 data points) from 3.8σ to 1.1 – 1.3σ (Banik & Kalaitzidis 2025)
- Inferred $H_0(z)$ curve agrees well with local void model (Mazurenko+ 2025; Jia+ 2023, 2025)
- Early time solutions to the Hubble tension face at least seven difficulties (Vagnozzi+ 2023)
- Ages of old Galactic stars and cosmic chronometers give *Planck* H_0 (Valcin+ 2025; Guo+ 2025)
- Very few proposed solutions to the Hubble tension had a different motivation and made a successful *a priori* prediction of a different phenomenon that is unlikely in Λ CDM.

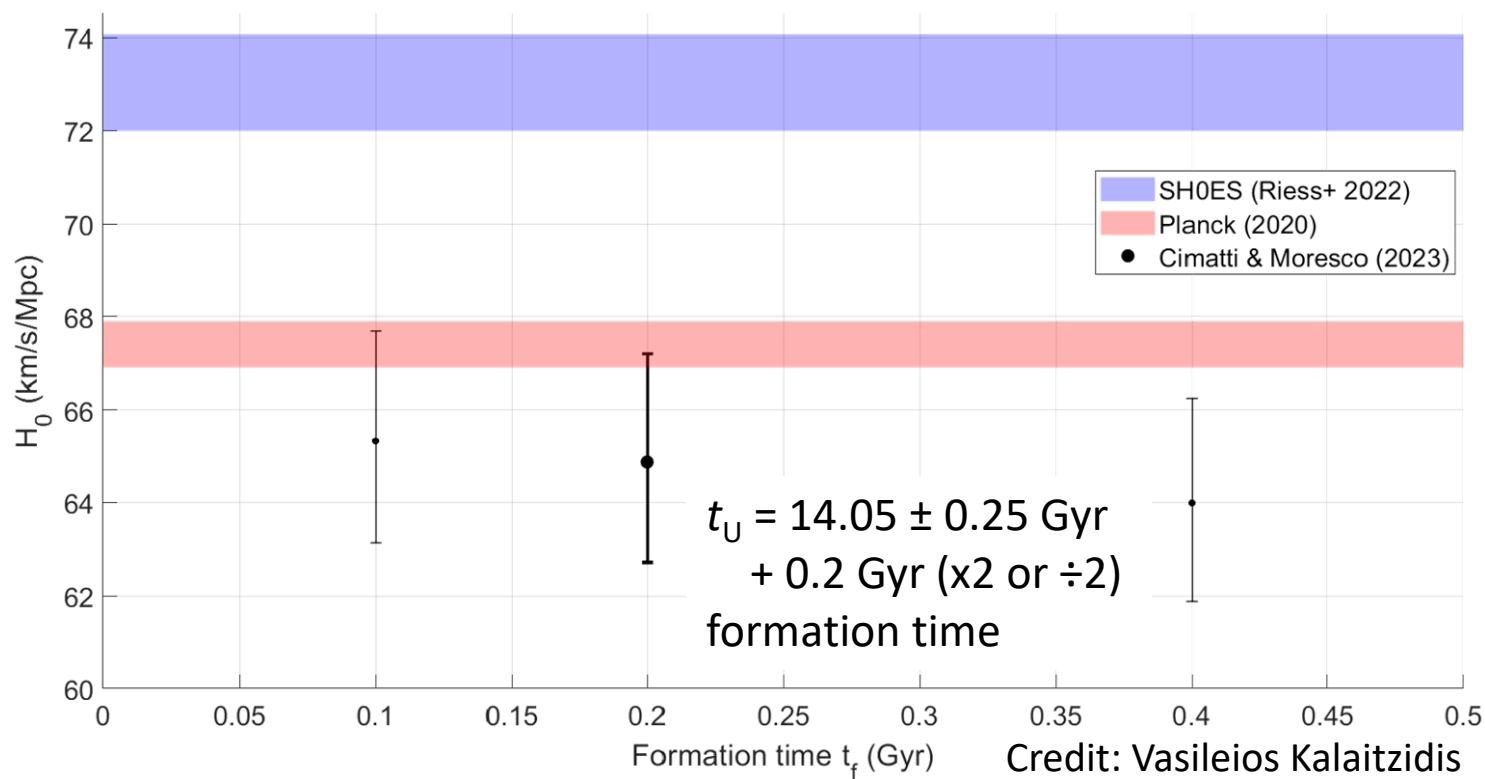
Dating the Universe

- The age of the Universe is inversely proportional to its expansion rate H_0

➤ The ages of 11 carefully chosen old objects (stars and globular clusters) give us H_0 independently of cosmological datasets

- $\Omega_m = 0.302 \pm 0.008$ is independent of CMB and contributes negligible uncertainty of 0.5 km/s/Mpc (Lin+ 2021)

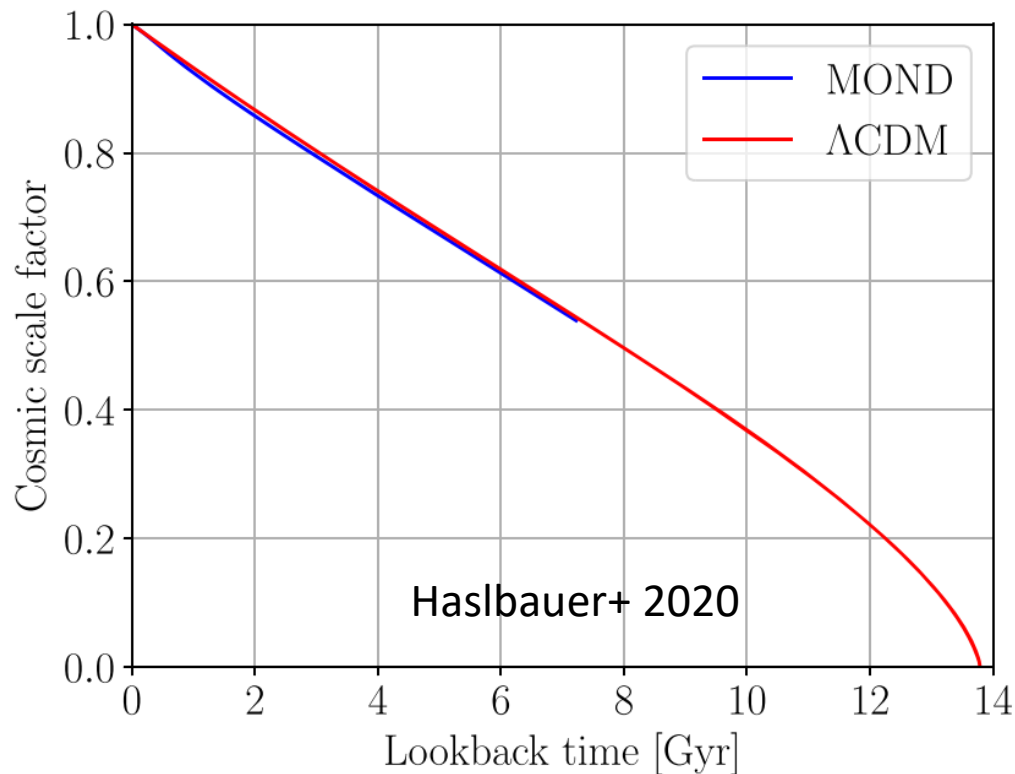
❖ *Planck* H_0 correct, so local dz/dr is inflated.



Credit: Vasileios Kalaitzidis
(RAS summer student)

A subtle change can solve the H_0 tension

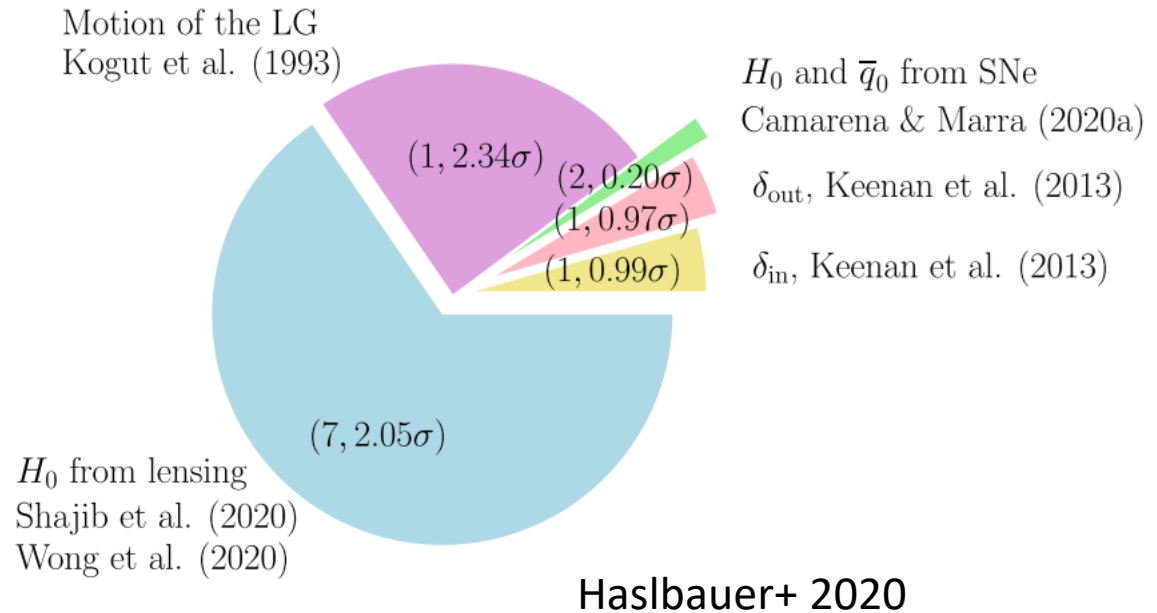
- A subtle difference between the actual and apparent expansion histories can solve the Hubble tension.
- Local void solutions create peculiar velocities out to quite high redshift in MOND due to slower decay of the gravity law
- This means the return to a *Planck* cosmology could be fairly slow, contradicting the assumptions in the famous Kenworthy+ 2019 paper which argued against the directly observed local void (*Planck* cosmology assumed to be fully recovered when $z > 0.1$).



Overall goodness of fit of local void model

- Overall tension is 2.53σ
- Pie chart: summary of individual contributions
- v_{LG} tension is based on fraction of void volume with a slower velocity in CMB frame
- Bulk flows not considered
- ❖ No substantial tensions.

(Degrees of freedom, equivalent tension for 1D Gaussian)

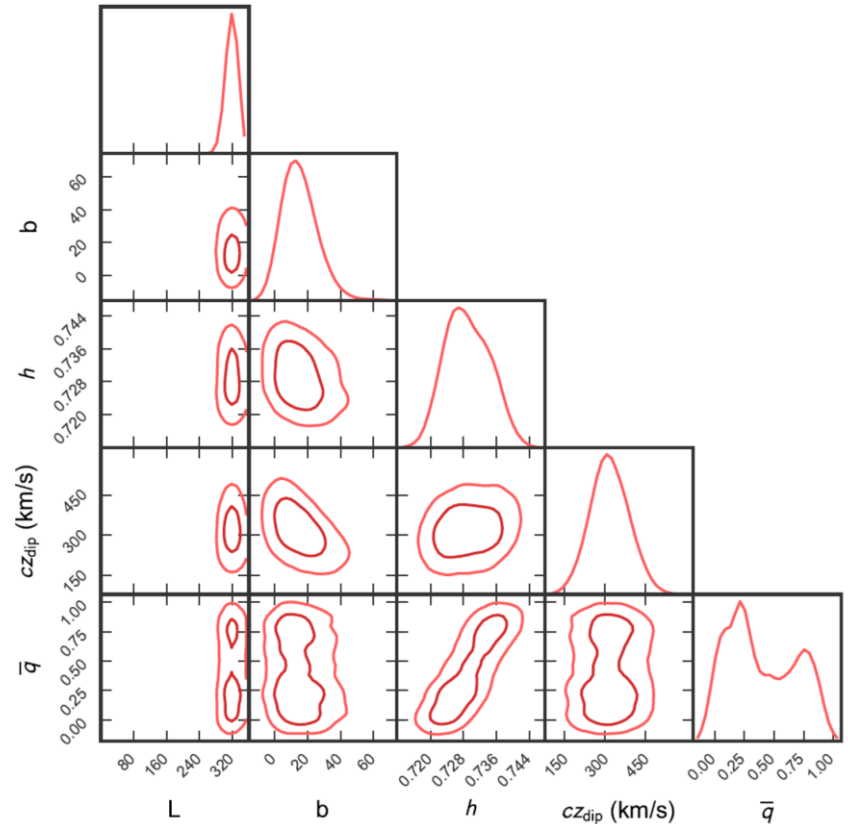


Dipole in supernova H_0

- Galaxy bulk flow reported by Watkins+ 2023 towards Galactic coordinates $(300^\circ, 0^\circ)$
- Supernovae over $z_{\text{CMB}} = 0.02 - 0.2$ analysed allowing for dipole in z and q_0 (common direction) such that:

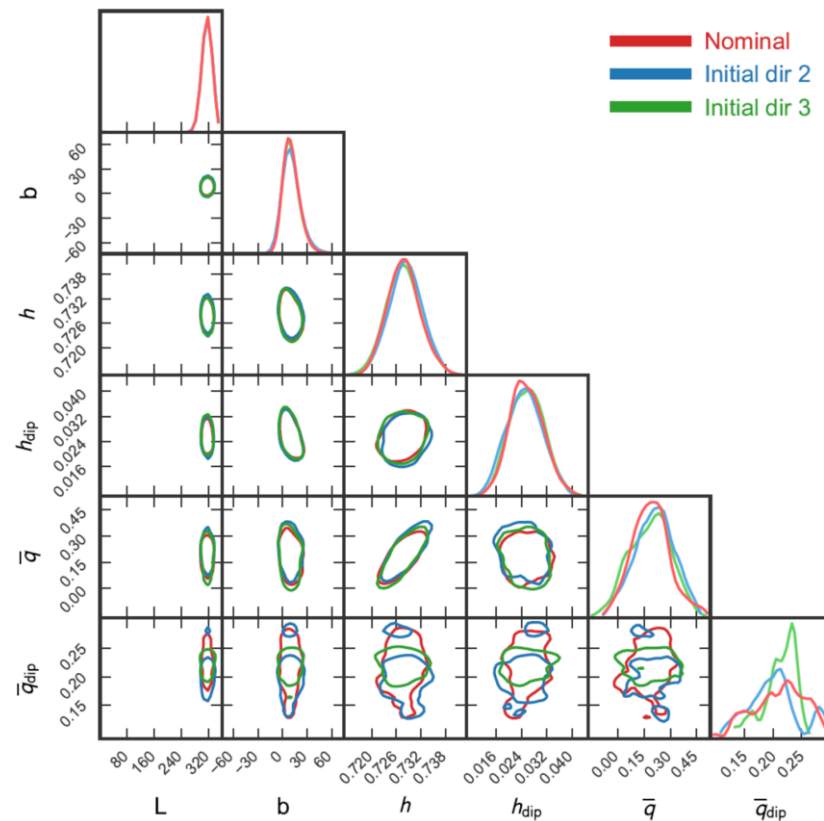
$$(1 + z) = (1 + z_c)(1 + \mathbf{z}_d \cdot \mathbf{n})$$

- Results from Pantheon+ SNe catalogue reveal a clear dipole in the Watkins+ 2023 direction
- Magnitude similar to their inferred bulk flow
- Plan is to compare SN peculiar velocity field (assuming *Planck* cosmology) with that predicted by the local void model (Mazurenko+, in prep).



Varying the initial direction for the MCMC

- Gradient descent used to minimise χ^2 starting from grid of initial directions
- Best outcome used for the nominal MCMC
- Two directions orthogonal to it & to each other also used as starting point for MCMC chain
- ❖ Results almost identical, so the MCMC result is barely sensitive to the initial direction used for the chain.
- ❖ A clear dipole signature is identified in a very well-defined direction, which is consistent with the independent determination using galaxies.



BAO observations over the last 20 years

- Grey points not used for χ^2
- At low redshift, often only D_V is available
- Some studies write an actual distance in Mpc rather than its dimensionless ratio with r_d . We looked up the assumed r_d .
- D_V often not given, even though D_c and D_H are available. We worked out D_V in these cases, combining fractional uncertainties in quadrature as D_V is essentially the geometric mean of D_c and D_H .

z	Survey	Reference	D_c	D_H	α_{sp}	D_V	Notes
0.068	Ho'leilana	Tully et al. (2023b)					α_s from individual structure at $z = 0.068^{+0.003}_{-0.007}$
0.097	6dFGS	Carter et al. (2018)					$r_d = 147.5$ cMpc
0.106	6dFGS	Beutler et al. (2011)					
0.11	SDSS DR17	de Carvalho et al. (2021)	•				
0.122	SDSS + 6dFGS	Carter et al. (2018)					$r_d = 147.5$ cMpc
0.15	SDSS DR7	Ross et al. (2015)					$r_d = 148.69$ cMpc
0.2	SDSS + 2dFGRS	Percival et al. (2007)					
0.2	SDSS DR7	Percival et al. (2010)					
0.24	SDSS DR12	Chuang et al. (2017)	•	•	♦		$r_d = 147.66$ cMpc; finer redshift bin
0.295	DESI DR1	DESI Collaboration (2024)					
0.32	SDSS DR9	Anderson et al. (2014)					$r_d = 149.28$ cMpc
0.32	SDSS DR12	Alam et al. (2017)					
0.32	SDSS DR12	Chuang et al. (2017)	•	•	♦		Coarser redshift bin
0.35	SDSS DR5	Percival et al. (2007)					
0.35	SDSS DR7	Percival et al. (2010)					
0.35	SDSS DR7	Chuang, Wang & Hemantha (2012)					
0.35	SDSS DR7	Chuang & Wang (2012)					
0.37	SDSS DR12	Chuang et al. (2017)	•	•	♦		Finer redshift bin
0.38	SDSS DR12	Alam et al. (2017)	•	•	♦		$r_d = 147.78$ cMpc
0.38	SDSS DR12	Ivanov et al. (2020)	•	•	♦		$r_d = 147.09$ cMpc
0.38	SDSS DR12	Schirra, Quartin & Amendola (2024)	•	•	♦		
0.44	WiggleZ Final DR	Blake et al. (2012)	•	•	♦		
0.49	SDSS DR12	Chuang et al. (2017)	•	•	♦		Finer redshift bin
0.51	SDSS DR12	Alam et al. (2017)	•	•	♦		
0.51	DESI DR1	DESI Collaboration (2024)	•	•	♦		
0.54	SDSS DR8	Seo et al. (2012)	•	•	♦		
0.57	SDSS DR12	Alam et al. (2017)	•	•	♦		$r_d = 147.78$ cMpc
0.57	SDSS DR12	Slepian et al. (2017)	•	•	♦		$r_d = 147.66$ cMpc
0.59	SDSS DR12	Chuang et al. (2017)	•	•	♦		Coarser redshift bin
0.6	WiggleZ Final DR	Blake et al. (2012)	•	•	♦		$r_d = 153.3$ cMpc (Blake et al. 2011)
0.61	SDSS DR12	Alam et al. (2017)	•	•	♦		$r_d = 147.78$ cMpc
0.61	SDSS DR12	Ivanov et al. (2020)	•	•	♦		$r_d = 147.09$ cMpc
0.61	SDSS DR12	Schirra et al. (2024)	•	•	♦		
0.64	SDSS DR12	Chuang et al. (2017)	•	•	♦		Finer redshift bin
0.7	DECaLS DR8	Sridhar et al. (2020)	•	•	♦		$r_d = 147.05$ cMpc (Planck Collaboration VI 2020)
0.7	SDSS DR16	Gil-Martin et al. (2020)	•	•	♦		
0.7	SDSS DR16	Zhao et al. (2021)	•	•	♦		Results identical to 2020 preprint
0.706	DESI DR1	DESI Collaboration (2024)	•	•	♦		
0.73	WiggleZ Final DR	Blake et al. (2011)	•	•	♦		
0.73	WiggleZ Final DR	Blake et al. (2012)	•	•	♦		$r_d = 153.3$ cMpc (Blake et al. 2011)
0.77	SDSS DR16	Wang et al. (2020)	•	•	♦		
0.8	SDSS DR14	Zhu et al. (2018)	•	•	♦		$r_d = 147.18$ cMpc (Planck Collaboration VI 2020)
0.81	DES Y1	DES Collaboration (2019)	•	•	♦		
0.835	DES Y3	DES Collaboration (2022)	•	•	♦		
0.85	SDSS DR16	de Mattia et al. (2021)	•	•	♦		
0.85	DES Y6	DES Collaboration (2024)	•	•	♦		
0.874	DECaLS DR8	Sridhar et al. (2020)	•	•	♦		$r_d = 147.05$ cMpc (Planck Collaboration VI 2020)
0.93	DESI DR1	DESI Collaboration (2024)	•	•	♦		
1.317	DESI DR1	DESI Collaboration (2024)	•	•	♦		
1.48	SDSS DR16	Neveux et al. (2020)	•	•	♦		Consensus BAO results (see also Hou et al. 2021)
1.491	DESI DR1	DESI Collaboration (2024)	•	•	♦		
2.33	DESI DR1	DESI Collaboration (2024)	•	•	♦		
2.33	DESI + SDSS	DESI Collaboration (2024)	•	•	♦		
2.33	SDSS DR16	du Mas des Bourboux et al. (2020)	•	•	♦		
2.4	DES Y1	DES & SPT Collaborations (2018)	•	•	♦		

Kinematic Sunyaev-Zel'dovich constraints

- Significant peculiar velocities implied by local void model would cause gas in galaxy clusters to boost CMB photons via kSZ effect
- KBC void implies about 20% underdensity out to 300 Mpc (deeper in redshift space)
- This is consistent with *Planck*
- Various common objections to local void model discussed in section 5.3 of Haslbauer+ 2020 (MNRAS, 499, 2845).

